EPIPOLAR GEOMETRY ESTIMATION FOR CONTOUR–BASED VISUAL SERVOING

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ABSTRACT

The problem of determining the epipolar geometry between two planar views is investigated. A novel Epipolar Geometry Estimation algorithm from apparent Contours (E.G.E.C.) is presented. EGEC searches for a minimum parameterization of epipoles position by the use of tangency lines at the apparent contours (or silhouettes) of unknown observed object. The retrieved epipoles are then used to design a visual servoing algorithm for mobile robots. Retrieving epipolar geometry from corresponding feature points is usually more involved than contours. However the estimation problem is hard to solve because the apparent contours of the projected scene objects can be of any shape. The existence and uniqueness of the estimated parameters is discussed and several simulations and experimental results are presented.

KEYWORDS: Epipolar Geometry, Visual-Servoing

1 Introduction

Visual servoing consists in steering the mobile robot toward a desired position known only through a previously acquired image (*desired image*) by a CCD camera mounted on. The steering control is based on features present in desired and in actual image [8, 5, 9]. Recently several authors approached the visual servoing problem making use of the epipolar geometry estimated from corresponding feature points [6, 1]. Epipolar geometry, indeed, relates two images of the same scene in a compact and mathematically tractable form [7]. However it may happen that the 3D scene does not exhibit feature points but only apparent contours [11]. Contours extracted from smooth image stream have been used to estimate the camera motion in [12].

This paper builds upon previous contribution [3, 2] and deals with the presentation of a novel algorithm for the estimation of epipolar geometry (EGEC) from a minimum number of generally shaped silhouettes. Existence and uniqueness of the solution is discussed for simple cases and through simulative and experimental evidence for scenes with generic objects.

2 Epipolar geometry and visual servoing law

Let z_c be the optical axis of a pin-hole camera mounted on a robot (Fig.1(a)) [7] and let $(X_a, Y_a, 0)$ be the position of camera/robot with respect to the *world-frame* $< wf >$. A point $\mathbf{X} = [X, Y, Z, 1]^T$ in $< wf >$, written in homogeneous notation, projects onto the

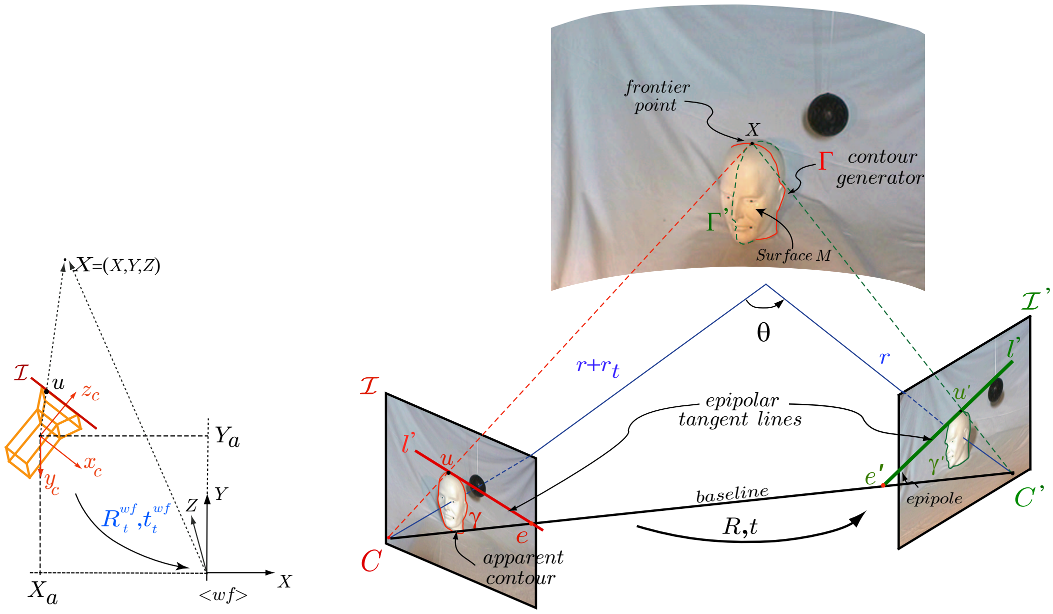

Figure 1: (a) Mobile robot reference frame with pin-hole camera mounted on; (b) Basic epipolar geometry notation.

point $\mathbf{u} = [u, v, 1]^T$ in the image plane $\mathcal{I}$ according to

$$k\mathbf{u} = \mathbf{K}[\mathbf{R}_c^{wf} \,|\, \mathbf{t}_c^{wf}]\mathbf{X}$$

where $k \in \mathbb{R}$ is an arbitrary scale factor, while $\mathbf{K}$, the camera calibration matrix [7], is supposed known and equal to $\mathbf{K} = [\alpha_x \; 0 \; u_0 \,; 0 \; \alpha_y \; v_0 ; 0 \; 0 \; 1]$. Suppose now that between two views $\mathcal{I}$ and $\mathcal{I}^{'}$ in C and C', a translation $\mathbf{t}$ and a rotation $\mathbf{R}$ occur. In this case $\mathbf{X}$ projects on two *corresponding points* $\mathbf{u} = [u, v, 1]^T$ and $\mathbf{u'} = [u', v', 1]^T$ (Fig. 1(b)).

Between these two views the *epipolar geometry* is defined and analytically described by the *fundamental matrix* $\mathbf{F} \in \mathbb{R}^{3\times 3}$ [7] that relates corresponding feature points through the *Correspondences Law*:

$$\mathbf{u}_i^{\prime T}\mathbf{F}\mathbf{u}_i = 0 \qquad \text{where} \qquad \mathbf{F} = \mathbf{K}^{-T}[\mathbf{t}]_\times \mathbf{R}\mathbf{K}^{-1} \tag{1}$$

The line between C and C' is referred to as the *baseline* (Fig. 1(b)) and intersects image planes in *epipoles* $\mathbf{e}$ and $\mathbf{e'}$. The plane through C, C' and $\mathbf{X}$ (called *epipolar plane defined by* $\mathbf{X}$) intersects $\mathcal{I}$ ($\mathcal{I}^{'}$) in the *epipolar line* $\mathbf{l} = \mathbf{F}^T\mathbf{u}_i'$ ($\mathbf{l}^{'} = \mathbf{F}\mathbf{u}_i$).

It is worthwhile to note that both epipoles and fundamental matrix, in the case of planar motion of two cameras, can be parameterized as a function of two variables $p \triangleq \frac{r+r_t}{r}$ and θ defined in the set $\Omega = \{p, \theta : p \in (0, +\infty), \theta \in (0, 2\pi)\}$, as in Fig.1(b) [2].

Consider now a textureless smooth 3D object as in Fig. 1(b) that projects on the actual (desired) image plane $\mathcal{I}$ ($\mathcal{I}^{'}$) in *apparent contours* γ ($\gamma^{'}$) defined as the projection of *contour generator* Γ ($\Gamma^{'}$) of the object on the image planes. The *frontier point* $\mathbf{X}$ is the intersection of Γ and $\Gamma^{'}$.Two very important properties are stated in the following [4]:

Property 1 *The epipolar plane tangent to the surface M of a non planar object intersects actual and desired image planes in two epipolar lines* $\mathbf{l}$ *and* $\mathbf{l'}$ *tangent to* γ *and* $\gamma^{'}$ *(Fig. 1(b)).*

Property 2 *The tangent line to apparent contour* γ *(*$\gamma^{'}$*) and passing through the epipole* $\mathbf{e}$ *(*$\mathbf{e}'$*), intersects the contour in* $\mathbf{u}$ *(*$\mathbf{u}'$*) that is the projection of the frontier point* $\mathbf{X} \in (\Gamma \cap \Gamma^{'})$*.*

The proposed general setup for the visual servoing consists of a translation of holonomic robot along the camera optical axis and a rotation around O toward C' (Fig.1(b)) [2, 3]. Both steps are based on a measure of u–coordinates epipole symmetry.

In the following we will present an algorithm to efficiently estimate the epipolar geometry from apparent contours of textureless surfaces. This is paramount in outdoor navigation where scene objects are highly unstructured and the search for feature points correspondences is a difficult task.

3 Epipolar geometry estimation from contours

In order to estimate the epipolar geometry it is fundamental to retrieve corresponding points [7]. However with contours it is not easy to extract correspondences. The approach here presented consists of using tangent lines to apparent contours all passing through a guessed epipole value in order to find the true epipole value verifying Property 2. The fundamental matrix and epipole parameterization discussed in Sec.2 for the planar motion case will play a key role in the optimization procedure as shown in the following Conjecture.

The optimization problem, that consists in minimizing the geometric distance between the tangency points and the epipolar lines, is stated in the following and the existence and uniqueness of a global minimum is investigated.

Conjecture 1 *A surface* M *projects itself on two apparent contours* γ *and* $\gamma^{'}$ *in the actual and desired cameras (Fig. 1) placed with given* (p^*, θ^*)*. Let* $\mathbf{F}(p, \theta)$*,* $\mathbf{e}(p, \theta)$ *and* $\mathbf{e}'(p, \theta)$ *be the parameterization discussed in Sec.2.*

For a given generic $(\hat{p}, \hat{\theta})$ *the line for* $\mathbf{e}(\hat{p}, \hat{\theta})$ *(*$\mathbf{e}^{'}(\hat{p}, \hat{\theta})$*) is tangent to* γ *(*$\gamma^{'}$*) in* $\mathbf{u}_i$ *(*$\mathbf{u}_i^{'}$*) Fig.1 (Note that* $\mathbf{u}_i$ *(*$\mathbf{u}_i^{'}$*) is a function of* $\hat{p}, \hat{\theta}$ *and of the contour itself). Given* $n \geq 2$ *tangency points* $\mathbf{u}_i$ *(*$\mathbf{u}_i'$*) then the optimization problem*

$$\min_{p,\theta} \sum_{i=1}^{n} w_i \left(d_i(p, \theta, \gamma, \gamma') \right)^2, \qquad n \geq 2 \tag{2}$$

where

$$w_i \triangleq \left(\frac{1}{{(\mathbf{F}\mathbf{u}_i)_1}^2 + {(\mathbf{F}\mathbf{u}_i)_2}^2} + \frac{1}{{(\mathbf{F}^T\mathbf{u}_i')_1}^2 + {(\mathbf{F}^T\mathbf{u}_i')_2}^2} \right) \tag{3}$$

and

$$d_i(p, \theta, \gamma, \gamma') \triangleq \mathbf{u}_i'^T(p, \theta, \gamma')\mathbf{F}(p, \theta)\mathbf{u}_i(p, \theta, \gamma) \tag{4}$$

admits an unique global point of minimum $(p_{min}, \theta_{min}) = (p^*, \theta^*)$*.* □

An analytical approach to (2) in a general context is intractable due to the contour shapes generality. Conjecture 1 is analytically discussed for a special object, a sphere, and experimentally validated for objects with any shape in [10]. It should be remarked that the conjecture states that only two tangency points are sufficient to guarantee the existence of the unique global minimum for the optimization problem (2).

4 Estimation algorithm

This section describes the epipolar geometry estimation algorithm that makes use of apparent contours and is based on the parameterization discussed in Section 2.

A large number of simulations and experiments showed different behaviors between the functional cost (2) with $w_i = 1$, given in Conjecture 1, and w_i as defined in (3). The functional cost with w_i as in (3) exhibits a low number of local minima which are more distinguishable from the global one, thus allowing to speed up the search for the global minimum. In the case of $w_i = 1$, (2) exhibits a large number of local minima close to the global minimum (p^*, θ^*), allowing to get it with higher accuracy. These experimental evidence suggested to design an estimation algorithm based on both the functional costs with $w_i = 1$ and w_i as in (3). The proposed heuristic algorithm is referred to as EGEC and is summarized in Table 4.

EGEC Algorithm
Epipolar Geometry Estimation from apparent Contours

1) Extract apparent contour γ_d from the desired image;
2) Extract the apparent contour γ_{a_1} from initial image $\mathcal{I}_1$;
3) Use a rough gridding algorithm for the optimization problem (2)-(3) to rapidly find an initial estimate $(\hat{p}_1, \hat{\theta}_1)$;

<u>for</u> $i = 2, 3, \ldots, n$
  4) Extract the contour γ_{a_i} from actual image $\mathcal{I}_i$ acquired during the visual servoing;
  5) Using $(\hat{p}_{i-1}, \hat{\theta}_{i-1})$ as an initial condition, use the *Levenberg-Marquardt algorithm* on (2) with $w_i = 1$ to find the point of minimum $(\hat{p}_i, \hat{\theta}_i)$ using tangent points at γ_{a_i} and γ_d.

<u>end</u>

Table 1: Algorithm for epipolar geometry estimation from contours (EGEC) .

EGEC, at each image measurement, finds the epipoles needed to implement the visual servoing strategy in Sec.2.

In steps 1),2) and 3) apparent contours γ_{a_1} and γ_d are extracted and a first estimation $(\hat{p}_1, \hat{\theta}_1)$ is obtained, near the global minimum. Then a more accurate *linear least squares* technique is applied during the robot motion (step 4) and 5)).

5 Experiments

Several experiments for the epipolar geometry estimation are presented. The experimental testbed consists of a CCD camera (HITACHI KP-D50) moving on a plane as described in Section 2, acquiring at 640×480. The computing system is an AMD Athlon at 1.4 GHz. Between *desired image* in C' and the *actual* one in C a large displacement also occurs. The observed scene consists of several non planar objects. Extraction of apparent contours was performed on MATLAB using median filtering and the Canny-edge detector.

Experiment 1 (one object) A set of images of a ball have been acquired by the camera moving on a circular trajectory. Results are reported for a set of five images. For

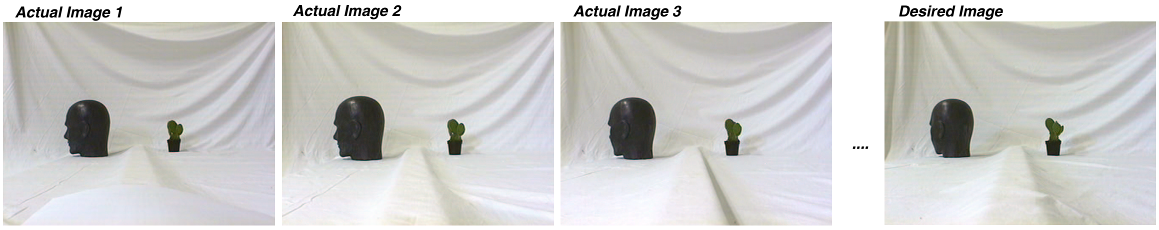

Figure 2: The first three current images and the desired one of Experiment 2.

each current image, parameters $\hat{p}$ and $\hat{\theta}$ have been estimated with EGEC and reported in Table 2(a).

image pair	(p^*, θ^*)	$(\hat{p}, \hat{\theta})$
$[1, D]$	$(1, 35°)$	$(0.99, 42°)$
$[2, D]$	$(1, 29°)$	$(0.99, 37.5°)$
$[3, D]$	$(1, 22°)$	$(0.99, 23°)$
$[4, D]$	$(1, 10°)$	$(0.99, 0°)$
$[5, D]$	$(1, 3°)$	$(0.98, 0°)$

(a)

image pair	(p^*, θ^*)	$(\hat{p}, \hat{\theta})$
$[1, D]$	$(1.1, 25°)$	$(1.11, 27.5°)$
$[2, D]$	$(1, 25°)$	$(1, 26.5°)$
$[3, D]$	$(1, 18°)$	$(1, 20.4°)$
$[4, D]$	$(1, 10°)$	$(1, 14.9°)$
$[5, D]$	$(1, 5°)$	$(0.98, 2°)$

(b)

Table 2: (a) Parameters estimation results for the Experiment 1; (b) Parameters estimation results for the Experiment 2.

In the first column pair of images (the actual and the desired D) is listed. The second and third columns report the true values for parameters p and θ and their estimation, respectively. EGEC exhibits a high accuracy for estimation of p, while $\hat{\theta}$ is not accurate when the angle becomes small. This is due to the fact that when θ approaches to zero the current contour generator tends to the desired one. In this case the frontier points become hill conditioned and the estimation algorithm based on (2) turns to be less accurate.

Note that at each step only two images are used to retrieve the epipolar geometry and that the images can be acquired also with a large displacement between cameras. The overall experiment took about 29 secs.

Experiment 2 (two generic objects) In the second experiment a head model and a cactus plant have been used. The images are acquired along the trajectory of the visual servoing strategy discussed in Section 2. For each i–th actual image (camera-robot in C_i), EGEC Algorithm (Table 4) has been used to search for global minimum of (2). The motion of current camera from C_1 toward C' is reported, together with acquired images and functional costs in Fig. 2, while in Table 2(b).

Comparing Experiment 1 and 2, it appears that performances obtained in the second are better than those got in the first one. This is due to the fact that here two different apparent contours are present in the images and then the functional cost (2) is built using four tangency points, and not two as in the first experiment, thus improving the overall performance of the estimation algorithm. The overall experiment took about 38 secs.

Several other experiments were addressed to show the validity of Conjecture 1.

6 Conclusions

In this paper the problem of determining the epipolar geometry between two views from apparent contours of objects in the scene has been approached. The framework is that of a camera-robot performing an image-based planar visual servoing. The visual servoing strategy is based on epipoles and refers to unknown 3D scene which do not exhibit any corner but only contours of texture-less surfaces. The estimation of the epipolar geometry is based on a special parameterization of planar displacements between the two cameras and is stated as an optimization problem. A conjecture has been presented for the existence and uniqueness of the global minimum. The conjecture has been validated both analytically (for simple objects) and experimentally. Work is in progress to extend the approach to full 3D camera motions.

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