Stability Analysis and Design of Multi-dimensional Haptic Systems

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ABSTRACT

This paper provides a convenient framework based on passivity and optimization for stability analysis and controller design for haptic systems involving multiple devices and human operators. The proposed approach exploits peculiar features of the multi-dimensional scenario and allows for taking into account structural constraints such as operator-device configuration and controller decentralization. A method for the design and implementation of structured controllers whose impact on the realism of interaction can be qualitatively tuned, is proposed.

Index Terms: Multi-contact, Virtual coupling design, Stability, Passivity.

1 INTRODUCTION

Stability is a key feature in haptic interaction with virtual environments, since unwanted oscillations can impair realism and, most importantly, may be potentially harmful for the human operator. The issue of stability in this context has been addressed by several authors since the early 90's [19] and involves quite a few aspects, since the systems at hand are complex and some of their components, namely the human operators, are difficult to model. Stability has been considered from multiple viewpoints, and passivity has often been exploited in this context, since it provides a powerful tool for analyzing etherogeneous interconnected systems [12]. The fundamental paper [9] and more recent works such as [22] provide different approaches to the characterization of passivity in sampled-data systems and in particular in haptics. In [16, 15, 17, 18] a discrete-time passivity framework is proposed to deal with stability analysis and controller (virtual coupling) design also in the presence of non-passive virtual environments, and in particular [17] addresses the presence of nonlinearities. In [20], an H_∞-type approach to the design of virtual couplings ensuring passivity and transparency is proposed. Most of the above contributions are mainly focused on the case of a single human operator interacting with a one-degree of freedom virtual environment, although also multivariable systems can be addressed to some extent.

Multi-contact interaction is an important issue in haptics [2]. Researchers from the computer haptics community, the branch of the haptic science closer to traditional robotics [21], have investigated several aspects in this scenario such as friction modeling [14, 1] and interaction with deformable objects [3], but mostly neglected stability issues. Recently, haptic systems where many users interact with a common virtual environment [10] and systems where a single human operator grasps virtual objects [13, 4] have been considered. The analysis of stability in the case of one or more human operators interacting with a shared virtual enviroment through multiple points of contact is a multidimensional problem that exhibits peculiar issues that need investigation.

In this paper, we deal specifically with stability analysis and controller design for haptic systems involving several human operators that interact with a common virtual environment through multiple devices. In particular, we extend some preliminary work presented in [5]. In that paper, the multi-contact scenario is addressed in a passivity-based framework which is related to the one employed in [16, 15, 17, 18], and several criteria based on linear matrix inequalities (LMI) [7] for both stability analysis and controller synthesis are proposed. Such approach suffers from a certain amount of conservatism in that it addresses multiple devices but does not distinguish between the different configurations in which one or several operators may interact with the devices themselves. As an example, the case of two haptic devices operated by a single human with two hands/fingers is considered the same as the case of two operators interacting separately with the same devices. Here, we propose a different and more general formulation of the problem that allows for reducing conservatism by introducing suitable relaxations that arise from taking explicitly into account the human-device configuration in the sense stated above. The proposed framework relies on the basic assumption that a human performing an interaction using m degrees of freedom be regarded as a $m-$input, $m-$output passive operator.

As far as controller (virtual coupling) design is concerned, particular attention has to be devoted to structural constraints that may arise. Indeed, multi-contact systems may be physically distributed, and therefore the virtual coupling may share only limited information with the devices and virtual environment due to decentralization and limited communication requirements.

As a first contribution, we provide a passivity-based stability criterion that takes into account the interaction constraints outlined above. Secondly, we propose an optimization-based method that exploits such stability condition for the design of stabilizing structured controllers. In particular, we focus on a virtual coupling class that represents a virtual spring-damper network that connects the endpoints of the environment according to an a-priori assigned structure (i.e., which endpoints are connected to which endpoints) that can reflect constraints such as decentralized implementation.

The paper is organized as follows. In Section 2 we report some preliminary specific results related to passivity and to modeling and passivity-based analysis of multi-contact multi-operator haptic systems; in Section 3 we derive the sought stability condition, which is then employed in Section 4 to derive the proposed virtual coupling design method. Section 5 reports an illustrative application example and conclusions are drawn in Section 6.

Notation

For a square matrix X, $X > 0$ ($X < 0$) denotes positive (negative) definiteness, X^T denotes transpose, and $\|X\|$ denotes some matrix norm of X; I_m is the $m \times m$ identity matrix. With $\mathscr{BD}(m;m_1,\ldots,m_N)$ we denote the set of $m \times m$ block-diagonal matrices whose N blocks have dimensions $m_1 \times m_1,\ldots,m_N \times m_N$, with

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$\sum_{i=1}^{N} m_i = m$. The latter notation is also used without ambiguity for block-diagonal transfer matrices of $m-$input, $m-$output linear systems and, more generally, of $m-$input, $m-$output operators. Finally, $X = \text{blockdiag}(X_1, \ldots, X_N)$ denotes a block-diagonal matrix with diagonal blocks $X_1, \ldots, X_N$.

2 PRELIMINARIES

2.1 Passivity results

The approach to stability analysis and controller design presented in this paper exploits a generalization of the passivity framework in [16]-[18], which is based upon the concepts of output strict passivity (OSP) and input strict passivity (ISP) [12, 8]. Let us recall the definitions of OSP and ISP in both the continuous and the discrete-time context.

Definition 1 (continuous-time passivity). Let Σ be a continuous-time dynamical system with input vector $u(t) \in \mathbb{R}^m$, output vector $y(t) \in \mathbb{R}^m$, and state vector $\psi(t) \in \mathbb{R}^n$. If there exists a continuously differentiable positive definite function $V(\psi) : \mathbb{R}^n \to \mathbb{R}$ (called the *storage function*) and $m \times m$ symmetric matrices Δ and Φ such that along all system trajectories $(\psi(t), u(t), y(t))$, $t \in \mathbb{R}$, the following inequality holds

$$\dot{V}(\psi(t)) < y(t)^T u(t) - y(t)^T \Delta y(t) - u(t)^T \Phi u(t),$$

then, system Σ is *passive* if $\Delta = \Phi = 0$, *output strictly passive with level Δ ($\Delta-$OSP)* if $\Delta > 0$, $\Phi = 0$, *input strictly passive with level Φ ($\Phi-$ISP)* if $\Delta = 0$, $\Phi > 0$, respectively.

Definition 2 (discrete-time passivity). Let Σ_d be a discrete-time dynamical system with input vector $u(k) \in \mathbb{R}^m$, output vector $y(k) \in \mathbb{R}^m$, and state vector $\psi(k) \in \mathbb{R}^n$. If there exists a positive definite function $V(\psi) : \mathbb{R}^n \to \mathbb{R}$ and $m \times m$ symmetric matrices Δ and Φ such that along all system trajectories $(\psi(k), u(k), y(k))$, $k \in \mathbb{N}$, the following inequality holds

$$\begin{aligned} \Delta V(\psi(k)) &= V(\psi(k+1)) - V(\psi(k)) \\ &< y(k)^T u(k) - y(k)^T \Delta y(k) - u(k)^T \Phi u(k), \end{aligned} \tag{1}$$

then the system is passive if $\Delta = \Phi = 0$, output strictly passive ($\Delta-$OSP) if $\Delta > 0$, $\Phi = 0$, input strictly passive ($\Phi-$ISP) if $\Delta = 0$, $\Phi > 0$, respectively.

Remark 1 Definitions 1 and 2, which specialize a more general concept of dissipativeness introduced in [11], differ from the commonly used notions of OSP/ISP in that the weights Δ and Φ are symmetric matrices instead of scalars. Such definitions are useful in order to address passivity problems in a MIMO context when structural constraints are present, as it will be clear in the following sections. Roughly speaking, since the passivity levels are closely related to energy dissipation, if the system at hand has a given structure, it is convenient to associate a level of dissipation to each block or component instead of the whole system. This is clearly a relaxation that is also allows for reducing conservatism when dealing with stability of interconnected systems via passivity arguments. Also, note that Δ and Φ need not necessarily be positive definite in this context: indeed, a dynamical system will be said to lack OSP (ISP) when the above definitions hold for non-positive definite Δ (Φ).

Let Σ_d be a discrete-time time-invariant linear system defined by the state space representation (A, B, C, D), where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{m \times n}$, $D \in \mathbb{R}^{m \times m}$. A straightforward extension of the standard Kalman-Yacubović-Popov lemma [8] applies.

Lemma 1 *System Σ_d is passive ($\Delta-$OSP, $\Phi-$ISP) if and only if there exists a symmetric matrix $P \in \mathbb{R}^n$ such that the following two matrix inequalities hold:*

$$P > 0$$
$$\begin{bmatrix} A^T PA - P + C^T \Delta C & A^T PB - \frac{C^T}{2} + C^T \Delta D \\ B^T PA - \frac{C}{2} + D^T \Delta C & B^T PB - \frac{D + D^T}{2} + D^T \Delta D + \Phi \end{bmatrix} < 0. \tag{2}$$

In order to address the design problems presented in this paper, we find it convenient to look for an alternative formulation of the above result in which some of the quantities involved, in particular matrices B, D, and Δ^{-1}, appear linearly in the matrix inequalities that define the passivity condition (2). This can be accomplished via a Schur complement argument, as the following result shows.

Lemma 2 *Let $\Delta > 0$. System Σ_d is passive ($\Delta-$OSP, $\Phi-$ISP) if and only if there exist a symmetric matrix $Q \in \mathbb{R}^n$ and a matrix $R \in \mathbb{R}^{(n+2m) \times n}$ satisfying the constraints*

$$\begin{array}{ll} \text{(a)} & Q > 0 \\[2mm] \text{(b)} & \begin{bmatrix} Q & R^T \\ R & S \end{bmatrix} > 0 \\[4mm] \text{(c)} & R = \begin{bmatrix} \frac{C}{2} \\ A \\ C \end{bmatrix} Q \end{array} \tag{3}$$

where

$$S = \begin{bmatrix} \frac{D^T + D}{2} + \Phi & B^T & D^T \\ B & Q & 0 \\ D & 0 & \Delta^{-1} \end{bmatrix}.$$

Proof. By pre- and post- multiplication of (3)(b) by the positive definite nonsingular matrix $\begin{bmatrix} Q^{-1} & 0 \\ 0 & I_{n+2m} \end{bmatrix}$ we get that (3)(b)-(3)(c) are equivalent to

$$\begin{bmatrix} Q^{-1} & \frac{C^T}{2} & A^T & C^T \\ \frac{C}{2} & \frac{D + D^T}{2} + \Phi & B^T & D^T \\ A & B & Q & 0 \\ C & D & 0 & \Delta^{-1} \end{bmatrix} > 0. \tag{4}$$

Taking the Schur complement with respect to the submatrix $\begin{bmatrix} Q & 0 \\ 0 & \Delta^{-1} \end{bmatrix} > 0$, (4) is in turn equivalent to

$$\begin{bmatrix} Q^{-1} & \frac{C^T}{2} \\ \frac{C}{2} & \frac{D + D^T}{2} + \Phi \end{bmatrix} - \begin{bmatrix} A^T & C^T \\ B^T & D^T \end{bmatrix} \begin{bmatrix} Q^{-1} & 0 \\ 0 & \Delta \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} > 0$$

which finally is equivalent to (2) once $P = Q^{-1} > 0$. ∎

2.2 Modeling and passivity analysis of multidimensional haptic systems

In this paper, we characterize multi-contact multi-operator haptic systems starting from the well-established framework of [9], [16]-[18]. In that framework, a haptic system is modeled as a sampled-data system (with sampling period T) resulting from the interconnection of four main components described by suitable I/O mappings (see Fig. 1): a human operator block H, a haptic device block D, a computer-simulated virtual environment E, and a virtual coupling V, whose role is to act as a controller in order to ensure the
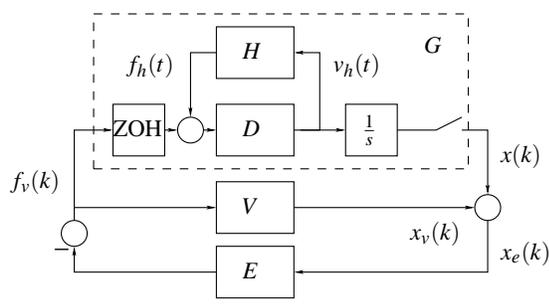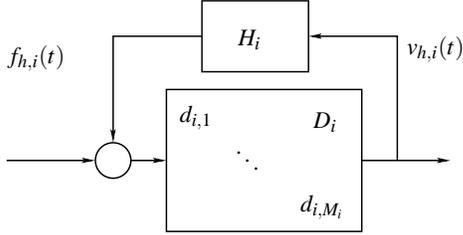
Figure 1: Haptic loop L

Figure 2: Interconnection of human operator H_i with haptic devices $d_{i,1},\ldots,d_{i,M_i}$

stability of the closed-loop. The mappings H and D are continuous-time, while E and V are described by discrete-time dynamical systems. Let L denote the overall loop.

In this paper, we fit the above framework to the case of N human operators H_i, $i = 1,\ldots,N$, each assumed to have m_i degrees of freedom (DOF). Each operator H_i is assumed to interact with M_i different devices denoted by $d_{i,j}$, $j = 1,\ldots,M_i$, where $d_{i,j}$ has $m_{i,j}$ DOF. Clearly, $m_i = \sum_{j=1}^{M_i} m_{i,j}$. All devices are coupled through a computer-simulated m−DOF virtual environment E (with $m = \sum_{i=1}^{N} m_i$) and through a virtual coupling V, which are both represented by m−input, m−output discrete-time dynamical systems. In order to simplify our exposition, we assume the absence of delay in the computations and consider only the impedance causality representation of the haptic system (see [15]), although the proposed results are believed to be easily extendable to cover both the delayed case and admittance causality.

The interaction of each operator H_i with the respective set of devices $d_{i,j}$, $j = 1,\ldots,M_i$ can be described by the feedback loop in Fig. 2, in which $f_{h,i}(t) \in \mathbb{R}^{m_i}$ represents the generalized force vector, $v_{h,i}(t) \in \mathbb{R}^{m_i}$ is the generalized velocity vector presented to the operator by the devices operated by H_i, and

$$D_i = \text{blockdiag}(d_{i,1},\ldots,d_{i,M_i}). \qquad (5)$$

It turns out that the overall system is described by the interconnection L in Fig. 1, where

$$
\begin{aligned}
H &= \text{blockdiag}(H_1,\ldots,H_N) \in \mathscr{BD}(m;m_1,\ldots,m_N) \\
D &= \text{blockdiag}(D_1,\ldots,D_N) \in \mathscr{BD}(m;m_1,\ldots,m_N)
\end{aligned} \qquad (6)
$$

$$
\begin{aligned}
f_h(t) &= [f_{h,1}^T(t) \ \cdots \ f_{h,N}^T(t)]^T \\
v_h(t) &= [v_{h,1}^T(t) \ \cdots \ v_{h,N}^T(t)]^T
\end{aligned} \qquad (7)
$$

and where $x(k) \in \mathbb{R}^m$ and $f_v(k) \in \mathbb{R}^m$ are the sampled generalized device displacement vector and sampled generalized force feedback vector, respectively.

Remark 2 Note that no peculiar structure is enforced a-priori on V and E. However, it is often the case that the haptic device and virtual coupling are lumped together. This requirement can be easily taken into account by assuming that V has a suitable block-diagonal structure as well. Clearly, additional requirements arising from decentralized computation and communication restrictions may enforce different contraints on V. For the sake of simplicity, in the sequel we will assume that V may only be constrained to be block-diagonal, even though the proposed approach is simply generalizable to a wide range of different structures.

Passivity-based stability analysis of haptic systems typically relies on the assumption that both the human and the device can be seen as passive operators; in particular, when an impedance causality model is employed, the device is assumed to be OSP to some extent [17]. The OSP level pertaining to a given device can be related to the amount of damping introduced into the system by the device itself. The problem of its computation has been addressed in [16] for linear devices via a robust stability argument, while in [18] it is shown, using a standard Lagrangian description of the device, that the OSP level can be related to dissipation in joint space. The latter results are easily generalizable to the OSP notion in Definition 1.
Motivated by the above observations, the following assumption is made in the context of this paper.

Assumption 1 (a) Each device $d_{i,j}$ is a $\Delta_{d_{i,j}}$−OSP continuous-time dynamical system, and (b) each human block H_i is a passive continuous-time m_i-input, m_i-output operator.

In view of Assumption 1, it is easily seen that the device block D defined by (5),(6) is Δ_D−OSP, where

$$\Delta_D = \text{blockdiag}(\Delta_{D_1},\ldots,\Delta_{D_N}) \qquad (8)$$

being

$$\Delta_{D_i} = \text{blockdiag}(\Delta_{d_{i,1}},\ldots,\Delta_{d_{i,M_i}}). \qquad (9)$$

In order to derive a stability criterion that takes into account the structure of the human-device block, we find it convenient to introduce a loop transformation parameterized by a diagonal matrix Γ whose form depends on the structure itself.
Let $\gamma_1,\ldots,\gamma_N$ be strictly positive scalars, and let Γ be the $m \times m$ diagonal matrix

$$\Gamma = \text{blockdiag}(\gamma_1 I_{m_1},\ldots,\gamma_N I_{m_N}). \qquad (10)$$

The further constraint

$$\text{trace}(\Gamma) = 1 \qquad (11)$$

can be enforced without loss of generality.
For any such Γ, consider the interconnected system L^Γ in Fig. 3 and denote

$$
\begin{aligned}
f_h^\Gamma(t) &= \Gamma f_h(t) = [\gamma_1 f_{h,1}^T(t) \ \cdots \ \gamma_N f_{h,N}^T(t)]^T \\
v_h^\Gamma(t) &= \Gamma v_h(t) = [\gamma_1 v_{h,1}^T(t) \ \cdots \ \gamma_N v_{h,N}^T(t)]^T.
\end{aligned} \qquad (12)
$$

The following result states some key properties of the loop L^Γ.

Theorem 1 *Let Assumption 1 hold. Then, the following properties pertain to L^Γ:*

(P1) *for any Γ of the form (10),(11), the block $H^\Gamma = \Gamma H \Gamma^{-1}$ is passive,*

(P2) *for any Γ of the form (10),(11), the block $D^\Gamma = \Gamma D \Gamma^{-1}$ is Δ_D−OSP, where Δ_D is as in (8),(9),*

(P3) *if there exists Γ of the form (10),(11) such that the loop L^Γ is stable, i.e., the signal $v_h^\Gamma(t)$ goes to zero in steady state, then so is the original loop L, i.e., the velocity vector $v_h(t)$ goes to zero in steady state.*
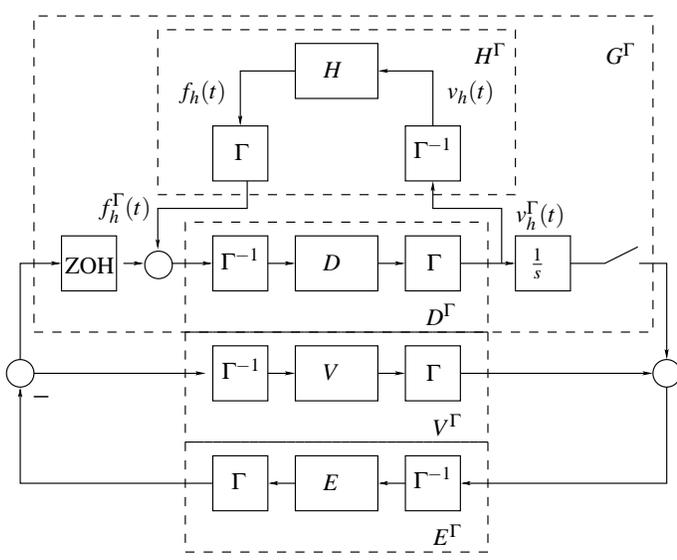
Figure 3: Equivalent haptic loop L^Γ

Figure 4: Transformed haptic loop $\hat{L}^\Gamma$

Proof. For each $i = 1, \ldots, N$, H_i is a continuous time passive operator by Assumption 1. Let $\psi_{H_i}(t)$ be the state vector of H_i. Then, there exists a positive definite storage function $V_{H_i}(\psi_{H_i})$ associated to each H_i such that $\dot{V}_{H_i}(\psi_{H_i}(t)) < f_{h,i}^T(t) v_{h,i}(t)$. Let $\psi_H(t) = [\psi_{H_1}^T(t) \ldots \psi_{H_N}^T(t)]^T$ be the state vector of H and consider the function

$$V_H^\Gamma(\psi_H) = \sum_{i=1}^{N} \gamma_i^2 V_{H_i}(\psi_{H_i}).$$

Clearly, V_H^Γ is positive definite for any Γ and moreover

$$\dot{V}_H^\Gamma(\psi_H(t)) < \sum_{i=1}^{N} \gamma_i^2 f_{h,i}^T(t) v_{h,i}(t) = f_h^{\Gamma T}(t) v_h^\Gamma(t).$$

Hence, $V_H^\Gamma(\psi_H)$ is a storage function for H^Γ. Property (P1) is therefore proved.
The proof of (P2) follows from a totally similar argument and is omitted.
Finally, the dynamics that relates $v_h(t)$ to $f_h(t)$ is clearly the same in L and L^Γ for any nonsingular Γ, hence (P3) is simply a consequence of the fact that any allowed matrix Γ is indeed nonsingular. ∎

The loop L^Γ introduced above and the related properties stated by Theorem 1 allow for analyzing stability of the haptic system by exploiting a generalization of the approach in [16]-[18]. To this purpose, consider the further transformed loop $\hat{L}^\Gamma$ in Fig. 4, where

$$\begin{aligned}
\hat{G}^\Gamma &= \frac{z-1}{Tz}[G^\Gamma + K] \\
\hat{V}^\Gamma &= \frac{z-1}{Tz}[V^\Gamma - K] \\
\hat{E}^\Gamma &= \frac{Tz}{z-1} E^\Gamma
\end{aligned} \tag{13}$$

being K a constant $m \times m$ matrix. The loop $\hat{L}^\Gamma$ is a purely discrete-time system in which the blocks $\hat{G}^\Gamma$, $\hat{V}^\Gamma$ and $\hat{E}^\Gamma$ can be characterized in terms of their OSP or ISP levels. The following result concerns the passivity properties of $\hat{G}^\Gamma$.

Theorem 2 *Let $\hat{G}^\Gamma$ be as in (13) and suppose Assumption 1 holds. Assume Δ_D as in (8),(9) and let*
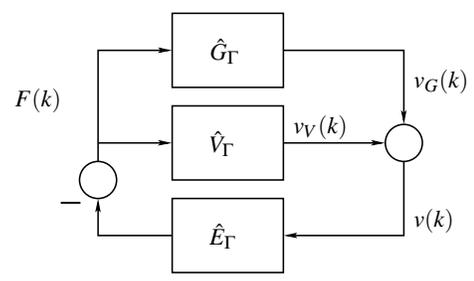
$$K = \frac{T}{2}\Delta_D^{-1}. \tag{14}$$

Then, for any Γ of the form (10),(11), $\hat{G}^\Gamma$ in (13) is (discrete-time) $\Delta_{\hat{G}}-OSP$ with

$$\Delta_{\hat{G}} = \frac{1}{T}\Delta_D. \tag{15}$$

Proof. Since D^Γ is Δ_D-OSP and H^Γ is passive for any Γ by Theorem 1, the result follows from a quite straightforward extension of Lemma 2 in [18] applied to G^Γ. ∎

3 LOOP STABILITY CRITERION

The characterization introduced in the previous section allows for deriving a sufficient stability condition for the haptic loop L via a simple generalization of the results in [17]. Indeed, since the loop transformation in (13) is the same as the one performed in [17], it is readily checked that if the discrete-time transformed loop $\hat{L}_\Gamma$ is asymptotically stable, then so is L^Γ, i.e., the signal $v_h^\Gamma(t)$ goes to zero in steady state. Hence, taking property 3 of Theorem 1 into account, we have the sought stability condition.

Theorem 3 *If there exists a matrix Γ of the form (10),(11) such that the transformed loop $\hat{L}^\Gamma$ is asymptotically stable, then the haptic loop L is stable, i.e., the generalized velocity vector $v_h(t)$ presented to the human operator(s) goes to zero in steady state.*

The problem now becomes that of assessing stability of the transformed loop $\hat{L}^\Gamma$. This can be done by exploiting the passivity levels of $\hat{G}^\Gamma, \hat{V}^\Gamma$ and $\hat{E}^\Gamma$. The following condition generalizes standard results concerning stability of feedback and parallel interconnections of passive systems and can be expressed in matrix inequality form.

Theorem 4 *If there exists a matrix Γ of the form (10),(11) and symmetric matrices $\Delta_{\hat{V}}$ $\Phi_{\hat{E}}$ $\Delta_{\hat{G}+\hat{V}}$ such that*

1. *$\hat{V}^\Gamma$ is $\Delta_{\hat{V}}-OSP$,*

2. *$\hat{E}^\Gamma + \Phi_{\hat{E}}$ is passive, i.e., $\hat{E}^\Gamma$ is $(-\Phi_{\hat{E}})-ISP$, and*

3. *the following matrix inequalities hold:*

$$\begin{aligned}
\Delta_{\hat{G}+\hat{V}} &> 0 \\
\Delta_{\hat{V}} &> 0 \\
\begin{bmatrix} \frac{1}{T}\Delta_D - \Delta_{\hat{G}+\hat{V}} & -\Delta_{\hat{G}+\hat{V}} \\ -\Delta_{\hat{G}+\hat{V}} & \Delta_{\hat{V}} - \Delta_{\hat{G}+\hat{V}} \end{bmatrix} &> 0 \\
\Phi_{\hat{E}} - \Delta_{\hat{G}+\hat{V}} &< 0,
\end{aligned} \tag{16}$$

then, the haptic loop L is stable.

Proof. Let $\psi_{\hat{G}}, \psi_{\hat{V}}, \psi_{\hat{E}}$ be the state vectors of $\hat{G}^\Gamma, \hat{V}^\Gamma, \hat{E}^\Gamma$, respectively. Since G^Γ is $\frac{1}{T}\Delta_D-$OSP and by conditions 1) and 2),

there exist positive definite storage functions $V_{\hat{G}}(\psi_{\hat{G}})$, $V_{\hat{V}}(\psi_{\hat{V}})$, and $V_{\hat{E}}(\psi_{\hat{E}})$ such that (see Fig. 4)

$$\Delta V_G(\psi_{\hat{G}}(k)) < F^T(k)v_G(k) - v_G^T(k)\frac{1}{T}\Delta_D v_G(k)$$
$$\Delta V_V(\psi_{\hat{V}}(k)) < F^T(k)v_V(k) - v_V^T(k)\Delta_{\hat{V}} v_V(k)$$
$$\Delta V_E(\psi_{\hat{E}}(k)) < -F^T(k)v(k) + v^T(k)\Phi_{\hat{E}} v(k)$$

Taking $V_{\hat{G}+\hat{V}}(\psi_{\hat{G}}, \psi_{\hat{V}}) = V_{\hat{G}}(\psi_{\hat{G}}) + V_{\hat{V}}(\psi_{\hat{V}})$ we get

$$\Delta V_{\hat{G}+\hat{V}}(\psi_{\hat{G}}(k), \psi_{\hat{V}}(k)) < F^T(k)v(k) - v^T(k)\Delta_{\hat{G}+\hat{V}} v(k)$$
$$- \begin{bmatrix} v_G^T(k) & v_V^T(k) \end{bmatrix} \begin{bmatrix} \frac{1}{T}\Delta_D - \Delta_{\hat{G}+\hat{V}} & -\Delta_{\hat{G}+\hat{V}} \\ -\Delta_{\hat{G}+\hat{V}} & \Delta_{\hat{V}} - \Delta_{\hat{G}+\hat{V}} \end{bmatrix} \begin{bmatrix} v_G(k) \\ v_V(k) \end{bmatrix}$$
(17)

and hence, by the third of (16)

$$\Delta V_{\hat{G}+\hat{V}}(\psi_{\hat{G}}(k), \psi_{\hat{V}}(k)) < F^T(k)v(k) - v^T(k)\Delta_{\hat{G}+\hat{V}} v(k)$$

i.e., the parallel interconnection of $\hat{G}^\Gamma$ and $\hat{V}^\Gamma$ is $\Delta_{\hat{G}+\hat{V}}$−OSP. Finally, taking $V(\psi_{\hat{G}}, \psi_{\hat{V}}, \psi_{\hat{E}}) = V_{\hat{G}+\hat{V}}(\psi_{\hat{G}}, \psi_{\hat{V}}) + V_{\hat{E}}(\psi_{\hat{E}})$ it holds that $V(\psi_{\hat{G}}, \psi_{\hat{V}}, \psi_{\hat{E}})$ is positive definite and, taking the fourth of (16) into account, we get

$$\Delta V(\psi_{\hat{G}}(k), \psi_{\hat{V}}(k), \psi_{\hat{E}}(k)) < 0$$

i.e., $V(\psi_{\hat{G}}, \psi_{\hat{V}}, \psi_{\hat{E}})$ is a Lyapunov function that proves asymptotic stability of $\hat{L}^\Gamma$. Therefore, the velocity vector $v_h(t)$ goes to zero in steady state by Theorem 3. ∎

4 STRUCTURED VIRTUAL COUPLING DESIGN

In this section, we exploit the stability condition outlined above to address the design of a virtual coupling V ensuring stability of the haptic loop. To this purpose, we first provide a computationally viable criterion for calculating OSP levels $\Delta_{\hat{V}}$ that ensure stability when the virtual environment is described as a linear time-invariant system. Then, we illustrate an LMI-based procedure for the design of a linear virtual coupling V of assigned structure such that the corresponding transformed virtual coupling achieves the level $\Delta_{\hat{V}}$ of OSP.

For ease of exposition, we assume the device parameters, i.e., the value of Δ_D, to be assigned a-priori, although the proposed procedure can be modified in order to address the design of the device dissipation levels to some extent (see [6]).

4.1 Computation of virtual coupling OSP level

Let (A_E, B_E, C_E, D_E) be a minimal state-space realization of a linear time-invariant virtual environment E. For given Γ, it is straightforward to check that a state space representation of the transformed virtual environment $\hat{E}^\Gamma$ is given by $(A_{\hat{E}}, B_{\hat{E}}(\Gamma), C_{\hat{E}}(\Gamma), D_{\hat{E}}(\Gamma))$ where

$$A_{\hat{E}} = \begin{bmatrix} A_E & B_E \\ 0 & I_m \end{bmatrix}, \quad B_{\hat{E}}(\Gamma) = \begin{bmatrix} B_E T \\ I_m T \end{bmatrix}\Gamma^{-1},$$
$$C_{\hat{E}}(\Gamma) = \Gamma\begin{bmatrix} C_E & D_E \end{bmatrix}, \quad D_{\hat{E}}(\Gamma) = \Gamma[D_E T]\Gamma^{-1}.$$

In view of Theorem 4, we now show that the computation of an OSP level $\Delta_{\hat{V}}$ ensuring stability can be cast as an optimization problem involving matrix inequality constraints. In this respect, a reasonable design criterion is the minimization of some norm of $\Delta_{\hat{V}}$ subject to the stability condition of Theorem 4.

Let us introduce the following optimization problem

$$\Delta_{\hat{V}} = \arg \min_{\Gamma, P, \Phi_{\hat{E}}, \Delta_{\hat{V}}, \Delta_{\hat{G}+\hat{V}}} \|\Delta_{\hat{V}}\|$$
$$\text{s.t.}$$
$$\Gamma \text{ is of the form (10),(11)}$$
$$\Delta_{\hat{V}} > 0$$
$$\Delta_{\hat{G}+\hat{V}} > 0$$
$$P > 0$$
$$\begin{bmatrix} A_{\hat{E}}^T P A_{\hat{E}} - P & A_{\hat{E}}^T P B_{\hat{E}}(\Gamma) - \frac{C_{\hat{E}}^T(\Gamma)}{2} \\ B_{\hat{E}}^T(\Gamma)P A_{\hat{E}} - \frac{C_{\hat{E}}(\Gamma)}{2} & B_{\hat{E}}^T(\Gamma)P B_{\hat{E}}(\Gamma) - \frac{D_{\hat{E}}(\Gamma)+D_{\hat{E}}^T(\Gamma)}{2} - \Phi_{\hat{E}} \end{bmatrix} < 0$$
$$\begin{bmatrix} \frac{1}{T}\Delta_D - \Delta_{\hat{G}+\hat{V}} & -\Delta_{\hat{G}+\hat{V}} \\ -\Delta_{\hat{G}+\hat{V}} & \Delta_{\hat{V}} - \Delta_{\hat{G}+\hat{V}} \end{bmatrix} > 0$$
$$\Phi_{\hat{E}} - \Delta_{\hat{G}+\hat{V}} < 0.$$
(18)

If a solution of (18) exists, then it yields a value of the level of OSP $\Delta_{\hat{V}}$ pertaining to $\hat{V}^\Gamma$ that satisfies the stability condition of Theorem 4, computed so as to minimize $\|\Delta_{\hat{V}}\|$. Note that the computation of both a suitable value of Γ and, according to Lemma 1, a suitable ISP level $-\Phi_{\hat{E}}$ of $\hat{E}^\Gamma$ that are compatible with the stability condition, is implicit in the solution of the above problem.

Remark 3 By performing suitable matrix manipulations, it is possible to cast (18) into the class of Bilinear Matrix Inequality (BMI) optimization problems, and as such its solution can be computed quite efficiently by means of available solvers.

4.2 Virtual coupling implementation

Once Γ and $\Delta_{\hat{V}}$ satisfing the stability condition have been obtained by means of (18), the problem becomes that of implementing a virtual coupling V such that the corresponding $\hat{V}^\Gamma$ is $\Delta_{\hat{V}}$−OSP, possibly taking structure constraints on V into account.

In order to provide a qualitative evaluation of the impact of the virtual coupling on the the sensation felt by the users during interaction, we follow the established practice of designing the virtual coupling as a discretized spring-damper system [17, 15]. Therefore, we consider a virtual coupling V of the form

$$V = V(z) = \left[\mathbf{K} + \mathbf{B}\frac{z-1}{Tz}\right]^{-1}$$
(19)

where $\mathbf{K} \in \mathbb{R}^{m \times m}$ and $\mathbf{B} \in \mathbb{R}^{m \times m}$ will be referred to as the stiffness and damping matrix, respectively. Clearly, the transfer matrix $V(z)$ in (19) models a network of interconnected virtual springs and dampers that connect the contact endpoints to each other according to the structure of matrices $\mathbf{K}$ and $\mathbf{B}$.

It is easily seen that the level $\Delta_{\hat{V}}$ of OSP pertaining to the transformed virtual coupling $\hat{V}^\Gamma = \hat{V}^\Gamma(z)$ can be made infinite provided that $\mathbf{B} = 0$ and $\mathbf{K} = \frac{2}{T}\Gamma^{-1}\Delta_D\Gamma$: indeed, in this case $\hat{V}^\Gamma(z)$ reduces to zero (see (13) and (14)). Nevertheless, in order to achieve this, $V(z)$ is constrained to have the same structure as Δ_D, and moreover it is not quite clear whether a virtual coupling of pure stiffness is desirable from the viewpoint of realism.

Our aim is to devise a strategy based on LMI optimization techniques, for the design of a virtual coupling of the form (19) satisfying the following requirements:

(R1) the transformed virtual coupling $\hat{V}^\Gamma$ is $\Delta_{\hat{V}}$−OSP, where $\Delta_{\hat{V}}$ and Γ are computed as the solution of (18), and therefore it gives rise to a stable closed loop,

(R2) $V(z)$ has an arbitrary a-priori assigned block-diagonal structure, i.e., $V(z) \in \mathscr{BD}(m; \bar{m}_1, \ldots, \bar{m}_{\bar{N}})$ for given $\bar{m}_1, \ldots, \bar{m}_{\bar{N}}, \bar{N}$, and

(R3) the relative amount of damping/stiffness introduced by the virtual coupling is enforced through the tuning of a free scalar parameter μ.

Note that the block-diagonal structure of V need not necessarily be related to the structure of H and D, although one may find it convenient to enforce a virtual coupling structure that reflects implementation constraints such as controller decentralization (see the example section).

To proceed, rewrite $V(z)$ in (19) as

$$V(z) = Tz\bar{V}(z)$$

where $\bar{V}(z)$ is given by

$$\bar{V}(z) = (zI - \bar{A})^{-1}\bar{B}$$

being

$$\bar{A} = (\mathbf{K}T + \mathbf{B})^{-1}\mathbf{B}, \quad \bar{B} = (\mathbf{K}T + \mathbf{B})^{-1}.$$

Conversely, we have

$$\mathbf{K} = \frac{1}{T}\bar{B}^{-1}(I - \bar{A}), \quad \mathbf{B} = \bar{B}^{-1}\bar{A}. \tag{20}$$

From (20), it turns out that the virtual coupling $V(z)$ reduces to pure stiffness in the limit case $\bar{A} \to 0$ and to pure damping for $\bar{A} \to I$. In particular, taking

$$\bar{A} = \mu I$$

where $0 < \mu < 1$, we get

$$\mathbf{K} = \frac{1}{T}(1 - \mu)\bar{B}^{-1}, \quad \mathbf{B} = \mu\bar{B}^{-1}. \tag{21}$$

It is then clear that the parameter μ enforces the relative amount of damping and stiffness of the virtual coupling, and that the transfer matrix $V(z)$ in (19) inherits its structure from that of $\bar{B}$, that is, if $\bar{B} \in \mathcal{BD}(m; \bar{m}_1, \ldots, \bar{m}_{\bar{N}})$, then $V(z)$ has precisely the same structure.

Given $\Delta_{\hat{V}}$, Γ and a scalar $0 < \mu < 1$, we now seek a matrix $\bar{B} \in \mathcal{BD}(m; \bar{m}_1, \ldots, \bar{m}_{\bar{N}})$ such that the corresponding $V(z)$ designed according to (19) and (21) satisfies the requirements (R1),(R2),(R3). In the sequel, we show that such $\bar{B}$ can be computed as the solution of an LMI feasibility problem.

The transformed virtual environment $\hat{V}^{\Gamma}$ has the transfer matrix

$$\hat{V}^{\Gamma}(z) = \frac{z-1}{Tz}[V^{\Gamma}(z) - K] = \frac{z-1}{Tz}[Tz\Gamma(zI - \bar{A})^{-1}\bar{B}\Gamma^{-1} - K]$$
$$= \frac{z-1}{Tz}[Tz(z - \mu)^{-1}\Gamma\bar{B}\Gamma^{-1} - K]. \tag{22}$$

After some manipulation, it is easily shown that $\hat{V}^{\Gamma}$ has a state-space realization $(A_{\hat{V}}, B_{\hat{V}}, C_{\hat{V}}, D_{\hat{V}})$ of order $2m$, where

$$A_{\hat{V}} = \begin{bmatrix} \mu I_m & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{\hat{V}} = \begin{bmatrix} \Gamma\bar{B}\Gamma^{-1}T \\ I_m \end{bmatrix}$$
$$C_{\hat{V}} = \frac{1}{T}\begin{bmatrix} (\mu - 1)I_m & K \end{bmatrix}, \quad D_{\hat{V}} = \begin{bmatrix} \Gamma\bar{B}\Gamma^{-1} - \frac{K}{T} \end{bmatrix} \tag{23}$$

Note that matrices $A_{\hat{V}}$ and $C_{\hat{V}}$ depend affinely on μ, and that $B_{\hat{V}}$ and $D_{\hat{V}}$ depend affinely on $\bar{B}$, while both Γ and $K = \frac{T}{2}\Delta_D^{-1}$ are constant at this stage.

The following result provides the sought virtual coupling design procedure.

Theorem 5 *Let $\Delta_{\hat{V}}$, Δ_D, Γ and $0 < \mu < 1$ be given. Consider the following LMI feasibility problem in the unknowns $\bar{B}$, R, and Q:*

(a) $\qquad\qquad Q > 0$

(b) $\qquad \begin{bmatrix} Q & R^T \\ R & S(\bar{B}) \end{bmatrix} > 0$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (24)$

(c) $\qquad R = \begin{bmatrix} \frac{C_{\hat{V}}}{2} \\ A_{\hat{V}} \\ C_{\hat{V}} \end{bmatrix} Q$

(d) $\quad \bar{B} \in \mathcal{BD}(m; \bar{m}_1, \ldots, \bar{m}_{\bar{N}})$

where

$$S(\bar{B}) = \begin{bmatrix} \frac{D_{\hat{V}}^T + D_{\hat{V}}}{2} & B_{\hat{V}}^T & D_{\hat{V}}^T \\ B_{\hat{V}} & Q & 0 \\ D_{\hat{V}} & 0 & \Delta_{\hat{V}}^{-1} \end{bmatrix}$$

being $A_{\hat{V}}, B_{\hat{V}}, C_{\hat{V}}, D_{\hat{V}}$ as in (23).
If a solution $\bar{B}^*, R^*, Q^*$ of (24) exists, then the virtual coupling $V = V(z)$ designed for $\bar{B} = \bar{B}^*$ according to (19),(21) satisfies the requirements (R1),(R2),(R3).

Proof. Conditions (24)(a), (24)(b) and (24)(c) imply that $\hat{V}^{\Gamma}$ is $\Delta_{\hat{V}}$-OSP by Lemma 2, and hence (R1) is accomplished. Moreover, (24)(d) yields (R2) since $V(z)$ preserves the structure of $\bar{B}$. The fact that μ represents the relative amount of damping/stiffness (R3) is clear from (21). $\blacksquare$

5 APPLICATION EXAMPLE

We consider a haptic system in which the interaction with a $2-$DOF linear virtual environment E is performed through a pair of $1-$DOF haptic devices d_x and d_y. Let the two devices be characterized by OSP levels Δ_{d_x} and Δ_{d_y}, respectively, according to the results in [18].
We are interested in performing stability analysis and virtual coupling design for the haptic loop in the following two cases:

i) both devices are operated by the same (2-DOF) human H_1, i.e., by a single human using both hands,

ii) the two devices are operated by two (1-DOF) humans H_1 and H_2.

According to the characterization in Section 2.2, each block of the haptic loop L is described by a $m-$input, $m-$output system with $m = 2$. Moreover, assumptions i) and ii) enforce the following problem structures, respectively.

i) $\begin{cases} N = 1, \ m_1 = 2, \ M_1 = 2, \ d_{1,1} = d_x, \ d_{1,2} = d_y \\ D_1 = \text{blockdiag}(d_{1,1} = d_x, d_{1,2} = d_y) \\ D = D_1 \\ H = H_1 \\ \Gamma = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \end{cases}$

ii) $\begin{cases} N = 2, \ m_1 = m_2 = 1, \ M_1 = M_2 = 1, \ d_{1,1} = d_x, \ d_{2,1} = d_y \\ D_1 = d_{1,1} = d_x, \ D_2 = d_{2,1} = d_y \\ D = \text{blockdiag}(D_1, D_2) \\ H = \text{blockdiag}(H_1, H_2) \\ \Gamma = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}, \quad \gamma_1, \gamma_2 > 0, \ \gamma_1 + \gamma_2 = 1. \end{cases}$

Let the virtual environment E be the backward Euler discretized version with sample period $T = 0.01 \ s$ of the mechanical system in Fig. 5, where $x_e = [x_{e,1}^T \ x_{e,2}^T]^T$ is the virtual environment displacement vector and $f_v = [f_{v,1}^T \ f_{v,2}^T]^T$ is the force feedback vector
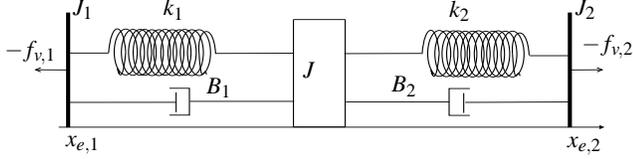
Figure 5: Example: Virtual environment model

		$\mu = 0.2$					
(L)	$\mathbf{K} =$	984.3	0	, $\mathbf{B} =$	2.4607	0	
		0	1425.7		0	3.5643	
(C)	$\mathbf{K} =$	977.4	77.2	, $\mathbf{B} =$	2.4435	0.1930	
		77.2	1407.9		0.1930	3.5197	
		$\mu = 0.7$					
(L)	$\mathbf{K} =$	392.68	0	, $\mathbf{B} =$	9.1627	0	
		0	670.52		0	15.6455	
(C)	$\mathbf{K} =$	384.15	40.76	, $\mathbf{B} =$	8.96	0.95	
		40.76	499.18		0.95	11.65	

Table 1: Example: Virtual coupling design solution

as depicted Fig. 1. Assume the parameter values $B_1 = 2$, $B_2 = -1.4$, $k_1 = 1$, $k_2 = 5$, $J = 0.1$, $J_1 = J_2 = 0.01$, in standard measurement units. Note that B_2 introduces negative damping.

A state space realization (A_E, B_E, C_E, D_E) of E can be readily obtained. The explicit expressions of A_E, B_E, C_E, D_E are not reported to save space.

We assume that the two haptic devices are characterized by different levels of OSP computed according to the results in [18]. Assume $\Delta_{d_x} = 0.1$ and $\Delta_{d_y} = 0.2$, yielding $\Delta_D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$.

Let us consider the problem structure corresponding to case i) (single operator). The *OSP* level $\Delta_{\hat{V}}$ of minimum $2-$norm that guarantees stability according to the proposed method can be computed by solving the optimization problem (18) for fixed $\Gamma = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$, yielding $\Delta_{\hat{V}} = \begin{bmatrix} 18.92 & 2.87 \\ 2.87 & 18.92 \end{bmatrix}$.

It is apparent that this solution is valid also for case ii) (two operators), since the structure of Γ pertaining to case i) is more restrictive than that of case ii).

The virtual coupling design can now be performed by computing the solution $\bar{B}$ of the LMI problem (24) once the relative amount of damping/stiffness μ is selected and possible structure constraints on V are enforced. We consider both the case of two 1-DOF controllers, each lumped to a device, and the case of one 2-DOF centralized controller. Clearly, the first case enforces a diagonal structure of $\mathbf{K}$ and $\mathbf{B}$ while the second does not. In Table 1 we report the values of the computed stiffness and damping matrices $\mathbf{K}$ and $\mathbf{B}$ corresponding to two different values of μ and both when a lumped (L) virtual coupling structure is enforced and when the structure is centralized (C).

We now introduce a perturbation on the virtual environment model E. In particular, we consider the system obtained by pre-multiplying the matrices C_E and D_E of the state space representation by the constant matrix $\begin{bmatrix} 1 & 0.3 \\ 0 & 1 \end{bmatrix}$.

For the perturbed environment model, it turns out the optimization problem (18) for case i) is not feasible, thus meaning that the proposed method fails to provide a virtual coupling OSP level that guarantees stability. On the contrary, a solution for case ii)

exists and is given by the pair $\Gamma = \begin{bmatrix} 0.532 & 0 \\ 0 & 0.468 \end{bmatrix}$, $\Delta_{\hat{V}} = \begin{bmatrix} 29.5977 & 0.5360 \\ 0.5360 & 29.6027 \end{bmatrix}$. In this case, the virtual coupling design procedure can be applied.

6 CONCLUSION

In this paper, the problem of stability assessment and virtual coupling design for haptic interaction systems involving multiple devices and human operators has been addressed, and a convenient framework based on passivity and optimization has been proposed. The main advantage of the approach is that it easily allows for taking into account the structural constraints that arise in a multi-dimensional scenario, in particular the different configurations in which one or several operators may interact with several devices. As far as virtual coupling design is concerned, a two-step procedure has been proposed that involves first the computation of the dissipation level that must be displayed by the virtual coupling in order to guarantee stability, and the subsequent implementation of a virtual coupling whose structure reflects possible constraints imposed on the control system.

REFERENCES

[1] F. Barbagli, A. Frisoli, K. Salisbury, and M. Bergamasco. Simulating human fingers: a soft finger proxy model and algorithm. In *Proc. Haptics Symposium*, Chicago, Illinois, March 2004.

[2] F. Barbagli, D. Prattichizzo, and J. Salisbury. *Multi-Point Physical Interaction with Real and Virtual Objects*, volume 18 of *STAR, Springer Tracks in Advanced Robotics*. Springer Verlag, Berlin Heidelberg, 2005.

[3] F. Barbagli, D. Prattichizzo, and K. Salisbury. A multirate approach to haptic interaction with deformable objects single and multipoint contacts. *International Journal of Robotics Research*, 4(29):703–715, September 2005.

[4] M. Bergamasco, A. Frisoli, and F. Barbagli. Haptics technologies and cultural heritage applications. *Computer Animation, 2002. Proceedings of*, pages 25–32, 2002.

[5] G. Bianchini, M. Orlandesi, and D. Prattichizzo. Analysis and design of multi-contact haptic systems: an LMI approach. In *Proc. of 46th IEEE Conference on Decision and Control*, pages 5761–5766, New Orleans, LA, USA, 2007.

[6] G. Bianchini, M. Orlandesi, and D. Prattichizzo. Analysis and design of multi-contact haptic systems: an LMI approach. 2007. Technical report, available online: http://www.dii.unisi.it/~giannibi/research/papers/lmihaptr.pdf.

[7] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear matrix inequalities in system and control theory*. SIAM, Philadelphia, 1994.

[8] C. Byrnes and W. Lin. Losslessness, feedback equivalence, and the global stabilization of discrete-time nonlinear systems. *IEEE Transactions on Automatic Control*, 39(1):83–98, 1994.

[9] J. Colgate and G. Schenkel. Passivity of a class of sampled-data systems: application to haptic interfaces. *Journal of robotic systems*, 14(1):37–47, 1997.

[10] M. Fotoohi, S. Sirouspour, and D. Capson. Stability and Performance Analysis of Centralized and Distributed Multi-rate Control Architectures for Multi-user Haptic Interaction. *The International Journal of Robotics Research*, 26(9):977, 2007.

[11] D. Hill and P. Moylan. Dissipative dynamical systems: basic input-output and state properties. *J. Franklin Inst.*, 309(5):327–357, 1980.

[12] R. Lozano, B. Brogliato, O. Egeland, and B. Maschke. *Dissipative systems analysis and control*. Springer Verlag, 2000.

[13] H. Maekawa and J. Hollerbach. Haptic display for object grasping and manipulating in virtual environment. *Robotics and Automation, 1998. Proceedings. 1998 IEEE International Conference on*, 3, 1998.

[14] N. Melder and W. S. Harwin. Improved rendering for multi-finger manipulation using friction cone based god-objects. In *Proc. Eurohaptics Conference*, pages 82–85, 2003.

[15] B. E. Miller. *Stability of haptic systems with nonpassive virtual environments*. Ph.D. Thesis, Nortwestern University, 2000.

[16] B. E. Miller, J. E. Colgate, and R. Freeman. Computational delay and free mode environment design for haptic display. In *ASME Dyn. Syst. Cont. Div.*, 1999.

[17] B. E. Miller, J. E. Colgate, and R. Freeman. Guaranteed stability of haptic systems with nonlinear virtual environments. *IEEE Transactions on Robotics and Automation*, 16(6):712–719, 2000.

[18] B. E. Miller, J. E. Colgate, and R. Freeman. On the role of dissipation in haptic systems. *IEEE Transactions on Robotics*, 20(4):768–771, 2004.

[19] M. Minsky, M. Ouh-Young, and O. Steele. Feeling and seeing: issue in force display. *ACM Trans. Comput.-Human Interactions*, 24(2):235–243, 1990.

[20] P. Naghshtabrizi and J. P. Hespanha. Designing transparent stabilizing haptic controllers. In *IEEE American Control Conference*, Minneapolis, MI, USA, 2006.

[21] J. Salisbury, F. Conti, and F. Barbagli. Haptic rendering: introductory concepts. *IEEE Computer Graphics and Animation*, 24:24–32, March 2004.

[22] S. Stramigioli, C. Secchi, A. J. van der Schaft, and C. Fantuzzi. Sampled data systems passivity and discrete port-hamiltonian systems. *IEEE Transactions on Robotics*, 21(4):574–586, 2005.