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Contact and Grasp Robustness Measures: Analysis and Experiments

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Abstract

In this paper we discuss some aspects related to the practical assessment of the quality of a grasp by a robotic hand on objects of unknown shape, based on sensorial feedback from tactile and force sensors on the hand. We briefly discuss the concept of contact and grasp robustness, pointing out that the former is an easily computable but overconservative sufficient condition for the latter. Some experimental results on a simple gripper, the so–called "Instrumented Talon", are reported as an illustration.

1. Introduction

This paper presents procedures for the assessment of the quality of grasps by robotic hands. The interest of having a good measure of the quality of the grasp is twofold: during planning of a manipulation sequence, it allows the optimization of the positioning of the hand with respect to the object to be grasped, and the grasping forces; during the execution of a grasping task, such measure can be used as a performance index according to which local optimization techniques can be used in order to react, at least sub–optimally, to external disturbances and modeling errors.

In the literature, there is a wide interest in the problem of planning good grasps. In [2] the quality criteria of grasp are based on the minimization of the sum of the maximum finger force (L_∞ metric) and of the total finger force (L_1 metric). In [4] the goodness of a grasp is defined in the space of object wrenches and is given as the radius of the largest closed ball, centered in the origin of the space, contained in the set of all the possible wrenches that can be resisted by applying at most unit forces at contacts. An optimality criterion depending on the specific task to be executed has been addressed by Li and Sastry in [5]. In [10] the approach to the analysis of the grasp quality consists of looking at the distance from the vector of contact forces to the nearest contact constraint, suggesting that the farther is the worst–case finger force from violation of a constraint, the better the grasp is. This approach is very intuitive and has been widely used in literature. Naturally, the choice of internal forces by the controller affects the grasp quality and one is led to consider, for each grasping configuration, a quality measure related to the best force distribution that an optimizing grasp force controller can possibly achieve.

In this paper, we build upon previous contributions by analyzing more closely two aspects that influence the concept of "good" grasp. Our analysis is performed in a quasi–static setting. Its first peculiarity is related with the fact that enveloping (alias "power", or "whole–hand") grasping is explicitly considered. In such style of grasping, not only the fingertips, but also the inner parts of the gripper are exploited in order to achieve a more robust hold on the object. This fact implies that contact constraints on the object may be imposed by members of the robotic hand which only enjoy limited mobility and are, therefore, not able to exert arbitrary forces at the contact at will. Secondly, it is observed that in most practical grasps, the set of contact constraints is redundant, in the sense that violation of some of them (slipping or detaching the contact) may well not imply mobilization of the object in the grasp. We therefore suggest that "contact robustness" measure is distinguished from "grasp robustness" measure, where the former is related to distances from the violation of any contact constraint, while the latter is concerned with actually overcoming the immobilization constraint of the object. It thus turns out that contact robustness is an easily computable but overconservative sufficient condition for grasp robustness, which is the property of actual concern in grasping.

Experimental activity on a testbed comprised of a simple enveloping gripper, the "Instrumented Talon", developed at the MIT AI Lab is finally described.

2. Contact Model

When the manipulation system is modeled by rigid-bodies, the i–th contact imposes that some components of the relative velocity between the surfaces are zeros. Mathematically, this can be written as $\mathbf{H}_i\ ({}^h\dot{\mathbf{c}}_i - {}^o\dot{\mathbf{c}}_i) = \mathbf{0}$, where $\mathbf{H}_i$ is a constant selection matrix depending on the physical model assumed for the i–th contact (cf. [12]) and ${}^h\mathbf{c}_i$, ${}^o\mathbf{c}_i$ are vectors locally describing the posture of reference frames attached to the surface of the hand and of the object, respectively. For the sake of simplicity, in this paper we only focus on hard–finger contact models. Small displacements of the contact frames can be expressed as a linear function of small displacements of the object $\delta\mathbf{u}$ and of the joints $\delta\mathbf{q}$, respectively, as $\delta^o\mathbf{c}_i = \tilde{\mathbf{G}}_i^T\ \delta\mathbf{u}$ and $\delta^h\mathbf{c}_i = \tilde{\mathbf{J}}_i\ \delta\mathbf{q}$. In juxtaposed vectorial notation, one has that rigid–body constraints can be summarized by the equation $\mathbf{H}(\tilde{\mathbf{J}}\delta\mathbf{q} - \tilde{\mathbf{G}}^T\delta\mathbf{u}) = 0$. The matrix $\mathbf{G} = \tilde{\mathbf{G}}\mathbf{H}^T$ is usually referred to as the "grasp matrix" (or "grip transform") while the matrix $\mathbf{J} = \mathbf{H}\tilde{\mathbf{J}}$ is called "hand jacobian".

As mentioned in the introduction, in this paper we allow for general grasping conditions, including enveloping grasps that exploit kinematically defective links to contact and constrain the object. Kinematic defectivity reflects in the fact that the hand jacobian is not full row rank. It has been shown in previous work of the authors [8] that, in enveloping grasping, the rigid body model in general is not adequate to describe unambiguously the system and in particular its force distribution problem. Moreover the rigid body model does not emphasize the dynamic of contact force control loops. Accordingly, a more accurate model describing how elastic energy can be stored in the system is necessary. We consider a simplified model of elasticity in the system, i.e. at each contact i we introduce a set of lumped "virtual springs" with characteristic stiffness $\mathbf{K}_{i_s}$. This allows us to describe small displacements of the contact force $\mathbf{t}_i$ from its equilibrium configuration as $\mathbf{t}_i = \mathbf{K}_{is}\mathbf{H}_i(\delta^h\mathbf{c}_i - \delta^o\mathbf{c}_i)$. Juxtaposing the n contact force vectors $\mathbf{t}_i$ in a single vector $\mathbf{t}$, one obtain: $\delta\mathbf{t} = \mathbf{K}_s(\mathbf{J}\delta\mathbf{q} - \mathbf{G}^T\delta\mathbf{u})$, where $\mathbf{K}_s = \mathrm{diag}(\mathbf{K}_{1s}, \ldots, \mathbf{K}_{ns})$.

Due to the unisense nature of contact forces and to friction, in order to avoid slippage and detachment of contacts, contact forces must satisfy unilateral constraints and the Coulomb's friction law. Letting t_{ik} $(k = x, y, z)$ be the component of the contact force $\mathbf{t}_i$ along the k–axis of the i–th contact frame (henceforth L_i) fixed to the object and chosen with the z–axis normal to the contact tangent plane, such constraints are written as

$$\text{a) } t_{iz} \geq 0, \qquad \text{b) } \sqrt{t_{ix}^2 + t_{iy}^2} \leq \mu_i t_{iz}, \tag{1}$$

where μ_i is the static coefficient of friction at the i-th contact.

3. Contact and Grasp Robustness

Suppose that a robotic hand grasps an object by means of n contacts and its configuration is of static equilibrium with balance equations: $\tau = \mathbf{J}^T\mathbf{t}$ and $\mathbf{w} = -\mathbf{G}\mathbf{t}$, where τ is the vector of joint torques and $\mathbf{w} = \left[\mathbf{f}^T, \mathbf{m}^T\right]^T$ is the external (disturbing) object wrench. We introduce the following hypotheses (cf. [8]):

H1: the subspace of *under–actuated* object displacements $\mathsf{ker}(\mathbf{G}^T)$ is empty;

H2: the manipulation system is asymptotically stabilized in the equilibrium point by a joint–position feedback controller with steady state gain $\mathbf{K}_p$;

H3: contact points do not change by rolling (this assumption is reasonable whenever the relative curvature is large).

Let us consider the vector

$$\mathbf{d}(\mathbf{t}) = \left[(d_{1c}, d_{1f}), \ldots, (d_{nc}, d_{nf})\right]^T, \tag{2}$$

where d_{ic} and d_{if} are the distances of $\mathbf{t}_i$ from the tangent plane to the object surface at the contact i and from the friction cone, respectively. The vector $\mathbf{d}(\mathbf{t})$ indicates how far the grasp is from violating contact constraints (1) and plays a fundamental role in the evaluation of the grasp quality. For instance, Kerr and Roth [10] base the quality measure of the grasp on the minimum component of the vector $\mathbf{d}(\mathbf{t})$.

3.1. Contact Robustness

In $\mathbb{R}^{3n}$, the inequality $\|\delta\mathbf{t}\| \leq \|\mathbf{d}(\mathbf{t})\|_\infty$ describes a sphere centered in the equilibrium contact force and provides a sufficient condition on the maximum euclidean norm of contact force perturbations $\delta\mathbf{t}$ in order to avoid slippage and detachment at all contacts. In order to assess the contact robustness of a grasp, the limitation of $\|\mathbf{d}(\mathbf{t})\|_\infty$ expressed in the contact force space, needs to be reflected in the space of external disturbances acting on the object. We will denote such disturbances as $\delta\mathbf{w}$ (referring to departures from the equilibrium condition). In the quasi–static setting chosen for this paper, the map from contact forces to object disturbance wrenches is $\delta\mathbf{w} = -\mathbf{G}\delta\mathbf{t}$ (via the principle of virtual work). However what is needed to assess contact robustness is the inverse of such map, namely the force distribution map from $\delta\mathbf{w}$ to $\delta\mathbf{t}$. As discussed e.g. in [7] we have, under (**H2**), that

$$\delta\mathbf{t} = -\mathbf{G}_K^R\, \delta\mathbf{w}; \qquad \text{with} \quad \mathbf{G}_K^R = \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}, \tag{3}$$

where $\mathbf{K} = (\mathbf{K}_s^{-1} + \mathbf{J}\mathbf{K}_p^{-1}\mathbf{J}^T)^{-1}$ is the composite grasp stiffness matrix (cf. [1]).

In order to make our following arguments independent from measurement units, we assume that the wrench vector $\delta\mathbf{w}$ is scaled with respect to the nominal value

$\delta \mathbf{w}_n$ of expected external disturbance wrenches in the task under consideration, such that $\delta \mathbf{w}$ is adimensional.

By using the inverse map (3), we can relate the limitation $\|\mathbf{d}(\mathbf{t})\|_\infty$ on contact forces $\delta \mathbf{t}$ with a limitation on the external disturbances $\delta \mathbf{w}$: the relationship $\delta \mathbf{t}^T \delta \mathbf{t} = \delta \mathbf{w}^T {\mathbf{G}_K^R}^T \mathbf{G}_K^R \delta \mathbf{w} \leq \|\mathbf{d}(\mathbf{t})\|_\infty^2$ describes an ellipsoid in the wrench space centered in zero and with principal axes $2\|\mathbf{d}(\mathbf{t})\|_\infty / \sigma_k(\mathbf{G}_K^R)$. Under the assumed hypotheses, the inscribed sphere with radius $\|\mathbf{d}(\mathbf{t})\|_\infty / \sigma_{max}(\mathbf{G}_K^R)$ represents a limit for the euclidean norm of $\delta \mathbf{w}$, ensuring that all contact constraints hold, notwithstanding the wrench disturbance. In other words, under the hypotheses **H1–H3** and in quasi–static conditions, a given grasp is able to resist any disturbance wrench $\delta \mathbf{w}$ without violating constraints (1), that is to say without detachment and slippage at any contact point, provided that

$$\|\delta \mathbf{w}\| \leq \frac{\|\mathbf{d}(\mathbf{t})\|_\infty}{\sigma_{max}(\mathbf{G}_K^R)}, \tag{4}$$

where $\sigma_{max}(\mathbf{G}_K^R)$ is the maximum singular value of the $\mathbf{K}$–weighted right–inverse $\mathbf{G}_K^R$, (3). Accordingly, the right–hand side term of (4) is defined as the **measure of contact robustness**. Note that condition $\mathsf{ker}(\mathbf{G}^T) = \emptyset$ (**H1**) guarantees that the inscribed sphere is 6–dimensional. A similar measure of contact robustness has been studied for non–defective manipulators in [11]. Our contribution here consists in pointing out the role of the complete stiffness matrix $\mathbf{K}$ in evaluating contact robustness for enveloping grasps. Furthermore, as observed in [5], the proposed measure is only a partial information on the grasp, since two ellipsoids may share the maximal inscribed sphere, though having different shapes.

An important aspect of grasping is that the contact force distribution can be modified by acting on internal forces, i.e. those self–balanced contact forces belonging to the nullspace of the grasp matrix $\mathbf{G}$. In the sequel we investigate how contact robustness may be improved by a redistribution of contact forces. In enveloping grasps the kinematic defectivity of the gripping mechanism may prevent the actual controllability of all the internal forces so that the best policy for redistributing contact forces in the grasp must be confined to modifying only some of the forces in $\mathsf{ker}(\mathbf{G})$. According to [7] and [8], the subspace of internal forces that are asymptotically reproducible (hence, quasi–statically controllable) is given by

$$\mathcal{F}_a = \mathsf{ker}(\mathbf{G}) \cap (\mathrm{im}(\mathbf{KJ}) + \mathrm{im}(\mathbf{KG}^T)) = \mathrm{im}(\mathbf{I} - \mathbf{G}_K^R \mathbf{G})\mathbf{KJ}. \tag{5}$$

Letting $\mathbf{E}$ denote a matrix whose columns span $\mathcal{F}_a$, the general solution to the balance equation $\mathbf{w} = -\mathbf{G}\mathbf{t}$ is given by $\mathbf{t} = -\mathbf{G}_K^R \mathbf{w} + \mathbf{E}\mathbf{y}$ where $\mathbf{y}$ parameterizes controllable internal forces in the basis $\mathbf{E}$.

It is an easy matter to show that $\|\mathbf{d}(\mathbf{t})\|_\infty$ can be infinitely improved by squeezing harder the object (i.e. by increasing the internal contact force). Therefore, in order to compare different grasps, upper bounds on the intensities of contact forces must be considered:

$$\|\mathbf{t}_i\| \leq f_{i,max} \qquad \text{with} \qquad f_{i,max} > 0. \tag{6}$$

Note that the inclusion of such upper bounds in grasp controller becomes necessary whenever we deal with limitations on actuator torques (i.e. for power expenditure) and fragility of the object and/or tactile sensors.

The following definition of contact robustness is based on previous arguments and consists in a maximization of the measure (4) on the subspace of reproducible internal forces: $\mathcal{F}_a = \mathsf{range}(\mathbf{E})$. By including in the distance vector $\mathbf{d}(\mathbf{t})$ new components, $d_{i,max} = f_{i,max} - \|\mathbf{t}_i\|$ (for all contacts), the **measure of potential contact robustness** is now defined as

$$\mathrm{PCR} = \max_{\mathbf{y}} \frac{\|\mathbf{d}(\mathbf{G}_K^R \mathbf{w} + \mathbf{E}\mathbf{y})\|_\infty}{\sigma_{max}(\mathbf{G}_K^R)}. \tag{7}$$

Let $\hat{\mathbf{y}}$ be the maximizing vector for the given upper bound $\mathbf{f}_{max}$ on $\|\mathbf{t}_i\|$. Any disturbance wrench $\delta\mathbf{w}$ such that $\|\delta\mathbf{w}\| \leq \mathrm{PCR}$ can be resisted without any contact slippage or detachment, provided that the (potential) internal force $\mathbf{E}\hat{\mathbf{y}}$ is actuated.

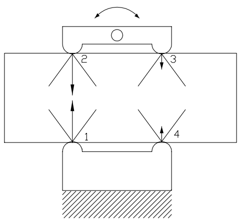

Figure 1: Four contacts grasp.

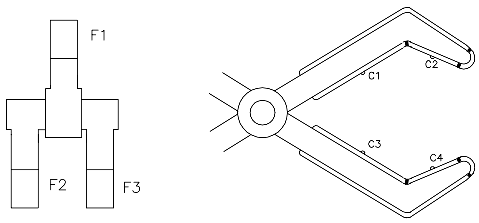

Figure 2: Instrumented talon.

3.2. Grasp Robustness

To illustrate the concept of grasp robustness, consider the planar grasp of fig. 1. Because of the contact forces at contacts 1 and 2, the grasp is intuitively firm and robust, although the minimum distance $\|\mathbf{d}(\mathbf{t})\|_\infty = d_{3f} = d_{4f} \approx 0$ and consequently the measure of contact robustness (4) is nearly zero. To obtain a less conservative estimate of how large an external disturbances can actually be resisted by the grasp, the fact that some of the contact constraints may be violated should be allowed, provided that a sufficient set of unviolated constraints remain to ensure immobilization of the object. We explicitly note that local slippage or contact detachment are possible because of the elasticity of bodies in contact, similarly to the theory of incipient slippage in classical contact mechanics. On the other hand, such elasticity is considered as lumped in virtual springs interposed at the contacts so that bodies still move as rigid bodies in space. The local details of friction and elasticity at the contacts may have large influence on the phenomena occurring in grasping under the above conditions, and they render an exact treatment very complex. In the following, we consider some simplifying assumptions that will allow a safe estimate of grasp robustness, with a degree of conservativeness however inferior to that of contact robustness estimates.

Our method is based on a set of simplified assumptions on the structure of the stiffness matrix $\mathbf{K}_{is}$ at the i–th contact that, consequently to the action of a disturbance $\delta\mathbf{w}$, can be in different contact states:
i) when, at the i–th contact, both constraints (1) are fulfilled, the corresponding stiffness matrix (in the local contact frame L_i fixed to the object at the contact i) is assumed diagonal and definite positive, ${}^{L_i}\mathbf{K}_{is} = \mathrm{diag}(K_{itx}, K_{ity}, K_{in})$;
ii) if the friction constraint (1–b) is violated, stiffnesses in the tangent plane are set to zero, i.e. ${}^{L_i}\mathbf{K}_{is} = \mathrm{diag}(0, 0, K_{in})$;
iii) if the contact condition (1–a) is violated at one contact point, the contact stiffness

at that point is assumed to be null: ${}^{L_i}\mathbf{K}_{i_s} = \mathbf{0}$.
Clearly, such assumptions conservatively disregard the fact that locally slipping contacts continue to contribute to the force balance.

For a given grasp consisting of n contact points, let $\mathcal{C}$ be the set of all possible combinations of the three contact states above (the cardinality of $\mathcal{C}$ is 3^n). For each grasp configuration $\mathcal{C}_j$ in $\mathcal{C}$, consider the global stiffness matrix $\mathbf{K}(\mathcal{C}_j) = (\mathbf{K}_s^{-1} + \mathbf{J}\mathbf{K}_p^{-1}\mathbf{J}^T)^{-1}$, where $\mathbf{K}_s = \text{diag}(\mathbf{K}_{1s}, \ldots, \mathbf{K}_{ns})$ and local stiffness matrices are defined according to the state of the corresponding contact in $\mathcal{C}_j$. Remember that $\mathbf{K}_{is} = {}^{L_i}\mathbf{R}^T\,{}^{L_i}\mathbf{K}_{is}\,{}^{L_i}\mathbf{R}$, where ${}^{L_i}\mathbf{R}$ is the rotation matrix of the contact frame L_i. Finally, the **measure of potential grasp robustness** (PGR) can be defined as

$$\max_{\mathcal{C}_j}\max_{\mathbf{y}} \quad \frac{\|\mathbf{d}(\mathbf{G}^R_{K(\mathcal{C}_j)}\mathbf{G}\mathbf{t} + \mathbf{E}(\mathcal{C}_j)\mathbf{y})\|_\infty}{\sigma_{max}(\mathbf{G}^R_{K(\mathcal{C}_j)})}, \tag{8}$$

$$\text{subject to} \quad \mathsf{ker}(\mathbf{K}(\mathcal{C}_j)\mathbf{G}^T) = \emptyset, \tag{9}$$

where $\mathbf{G}^R_{K(\mathcal{C}_j)}$ and $\mathbf{E}(\mathcal{C}_j)$ are the weighted pseudoinverse (3) and the basis matrix of asymptotically reproducible internal forces (5), respectively, evaluated with $\mathbf{K}(\mathcal{C}_j)$ modified as described above. Note that condition (9) implies that candidate grasp configurations $\mathcal{C}_j$'s need only be considered among those that can actually immobilize the object, thus effectively reducing the dimension of the set to be searched for the highest contact robustness.

In a few words, such a measure is similar to measure (7) but here we take into account the fact that some of the contact constraints (as friction constraints of contacts 3 and 4 in fig. 1) may be violated without that the grasp of the object fails. The overconservativeness of contact robustness (7) with respect to grasp robustness (8) will be illustrated by the following experiment.

4. Experiment

Grasp analysis tools discussed in section 3 have been employed in an experimental testbed consisting of a simple one–degree-of–freedom gripper or "Instrumented Talon" developed, in part, through collaboration with Harvard University for use on the M.I.T. Whole–Arm Manipulator [13]. The talon (fig.2) has three fingers, each of them equipped with four tactile–sensitive piezoelectric pads [3], and strain–gage based force sensors at the base of the fingers. In its present version, the instrumented talon is only able to sense forces in the finger plane.

The instrumented talon shares its computational resources with the robotic system it is a part of. The complete computational architecture consists of five Motorola 68040 single board computers working in parallel on a real–time software environment. At present, sensory data from the talon are acquired through an interface board and two fiber optic lines to a 68040 VME. The bulk of grasp analysis computations are carried over by a second 68040 board.

In a first experiment, the talon was used to grasp a 1 Kg parallelepiped (see fig. 3). For this experiment, the stiffness of the finger position controller was set very high. The composite grasp stiffness matrix $\mathbf{K}$ results with $\mathbf{K}_p \to \infty$ as $\mathbf{K} \approx \mathbf{K}_s$. Moreover, due to the homogeneity of materials used, we assume that the stiffness matrix has a very simple form: $\mathbf{K} = k\mathbf{I}$, where k is scalar. It can be easily shown that k does not influence the grasp analysis of previous sections.

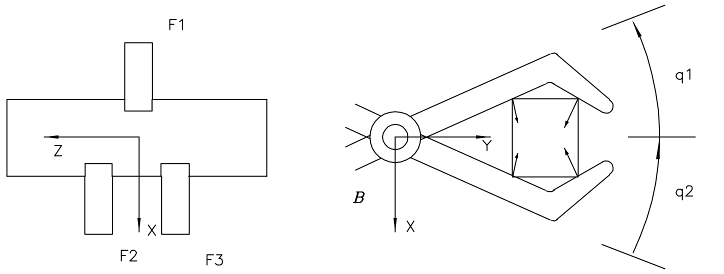

Figure 3: Talon grasping.

In fact, under the condition $\mathbf{K} = k\mathbf{I}$, the weighted pseudoinverse (3) together with the basis matrix of (5) do not depend upon k, and consequently measures (4, 7 and 8) do not depend upon k as well. The friction coefficient for the considered contact conditions is ca. 0.78. For the grasp under consideration (fig. 3), joint angles are $q_1 = 35$ deg and $q_2 = -35$ deg, and the constraint distance vector (2) is $\mathbf{d}(\mathbf{t}) = [(0.1, 0.002)\ (0.47, 0.2)\ (2, 0.1)\ (2.8, 1.5)\ (2, 0.12)\ (2.8, 1.5)]^T$ (in N), showing that the contact c_1 is closest to slippage. In this case, the measure of contact robustness (4) is $\|\mathbf{d}(\mathbf{t})\|_\infty / \sigma_{max}(\mathbf{G}_K^R) = 0.002/26.2 = 0.000076$, where $\delta\mathbf{w}$ is scaled with respect to $[(1,1,1)\mathrm{N}, (1,1,1)\mathrm{Nm}]$. In order to evaluate how much the grasp can be improved by modifying internal forces, it is necessary to determine the subspace of asymptotically reproducible internal forces. Note that, in this example, while $\dim \mathsf{ker}(\mathbf{G}) = 12$, we have that $\dim \mathcal{F}_a = 1$. In particular, contact forces in $\mathcal{F}_a$ are depicted in fig. 3.

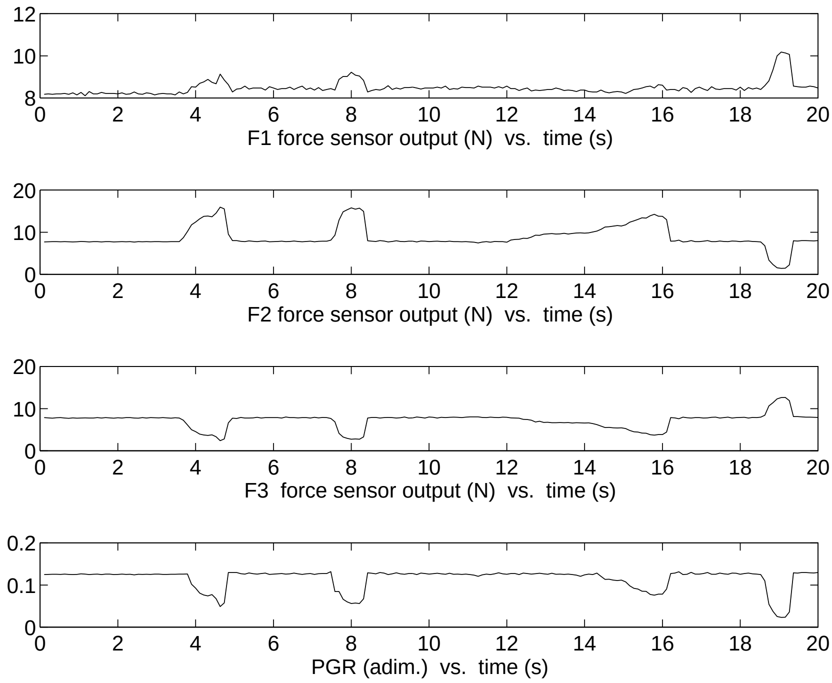

Figure 4: Force sensor outputs and estimated PGR corresponding to application of four external disturbance on the grasped object.

The algorithm described in [9] for real time evaluation of the potential contact robustness measure (7) (having set $f_{i,max} = 20\mathrm{N}$) was implemented on a devoted 68040 processor, yielding a rate of 10 KHz, while sensor measurements were processed at a faster rate to estimate the external wrench $\mathbf{w}$. The potential contact robustness (PCR) for this grasp (obtained through averaging because of sensor noise) was $\overline{\mathrm{PCR}} = 0.002$, thus showing the beneficial effect of increasing internal forces.

Finally, we consider the measure of potential grasp robustness (PGR). For the equilibrium grasp configuration of fig. 3, the (averaged) value of $\overline{\mathrm{PGR}} = 0.1594$ was obtained, corresponding to the grasp state when the innermost contacts on the three fingers are supposed to locally slip. Note that $\overline{\mathrm{PGR}} \approx 80\overline{\mathrm{PCR}}$. The measure of potential grasp robustness has been evaluated in real time while unknown disturbances were manually applied to the object grasped in two different configurations. Results are reported in fig. 4, showing the decrease of the PGR corresponding to the increase of the disturbing action on the object.

5. Conclusions

We have investigated the robustness of robotic grasping with respect to external disturbances in a more general framework than previous works on the same topic. Namely, we considered the case when the gripper is kinematically defective (as happens in simple grippers and in whole–arm mechanisms), and we underscored the difference between contact robustness and grasp robustness. Some preliminary experimental results have been presented, indicating the viability of the proposed tools for real–time implementation of optimizing force policies in grasping.

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