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Motion-decoupled internal force control in grasping with visco-elastic contacts

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Abstract

Robotic grasps exhibiting visco-elastic contact interactions with the manipulated object are considered. Control of internal forces is investigated. The presence of non-negligible compliance at contacts, implies that the object dynamics cannot be neglected when attempting to control internal forces without affecting the object position. A dynamic internal force control is proposed. It is decoupled with respect to the rigid-body object motions.

1 Introduction

This paper deals with robotic grasps where contacts are modeled through visco-elastic interactions. This is the case of hands with soft fingers [13] or that of advanced medical applications.

In [3, 9], it has been shown that the use of visco-elastic contact model may result mandatory to model the force distribution for kinematically defective grasps, i.e. those grasps with a non-trivial nullspace of the Jacobian transpose. Kinematically defective grasps may lead to hyperstaticity and as a consequence the rigid-body model of contact interactions would not be able to solve the force distribution problem.

In [12], authors investigated the problem of planning joint motions to reposition and reorient the object within whole–arm grasps. The grasp robustness by kinematically defective devices has been studied in [14, 11]. In [5], authors analyzed the stability of enveloping grasps using a model of the contacts and joints compliance. In [4] the rigid–body kinematics of general manipulation systems and their manipulability have been investigated. Metrics for grasp goodness taking into account geometry of the grasp and computational complexity of algorithms were studied in [7]. In [6] authors analyzed motion of objects subject to contact constraints.

This paper focuses on the internal force control of grasps

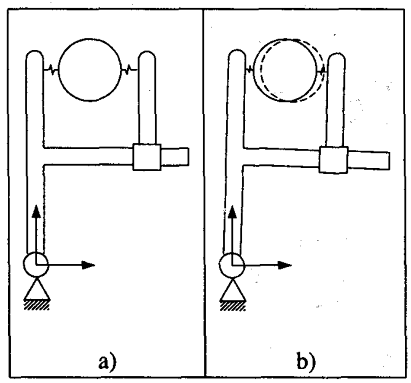

Figure 1: A 2 DoF's manipulation system. Squeezing action of the prismatic joint.

with general kinematics including kinematically defective, redundant and hyperstatic grasps. The class of grasps with general kinematics will be formalized in Section 2, its distinctive feature is that grasps with contacts on fingers' inner parts and/or with fingers having a low number of DoF's are allowed.

Internal forces play a key role in grasp control. If the manipulation system cannot be modeled by rigid-body contacts, the object dynamics cannot be ignored to design the internal-force controller. As an example consider the 2 DoF's manipulation system in fig.1-a). A control policy ignoring object dynamics, for instance a force step on the prismatic joint, squeezes the manipulated object but an undesired and dangerous transient motion of the object arises.

The aim of this work is to fix such a problem. A feedback controller which decouples the internal force control loop from the object dynamics is proposed.

2 Dynamic model

This paper is based on linearized dynamics of grasps with general kinematics derived in [9, 10].

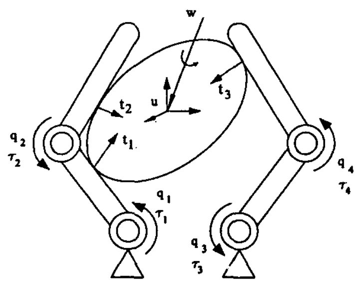

Figure 2: Vector notation for grasp with general kinematics.

Denote by $\mathbf{q} \in \mathbb{R}^q$ the vector of manipulator joint positions, $\tau \in \mathbb{R}^q$ the vector of joint actuator torques, $\mathbf{u} \in \mathbb{R}^d$ the vector locally describing the position and the orientation of a frame attached to the object, and $\mathbf{w} \in \mathbb{R}^d$ the vector of forces/torques resultant from external forces acting directly on the object, cf. fig. 2. In the literature, $\mathbf{w}$ is usually referred to as the disturbance vector.

The force/torque interaction $\mathbf{t}_i$ (fig. 2) at the i-th contact is taken into account by using a lumped-parameter ($\mathbf{K}_i$, $\mathbf{B}_i$) model of visco-elastic phenomena. The contact force vector $\mathbf{t}_i$ is

$$\mathbf{t}_i = \mathbf{K}_i({}^h\mathbf{c}_i - {}^o\mathbf{c}_i) + \mathbf{B}_i({}^h\dot{\mathbf{c}}_i - {}^o\dot{\mathbf{c}}_i) \qquad (1)$$

where vectors ${}^h\mathbf{c}_i$ and ${}^o\mathbf{c}_i$ describe the postures of two contact frames, the first on the manipulator and the second on the object, where the i-th contact spring and damper are anchored. To compact notation, contact forces and contact points are grouped into vectors $\mathbf{t}$, ${}^h\mathbf{c}$ and ${}^o\mathbf{c}$. Similarly, $\mathbf{K}_i$'s and $\mathbf{B}_i$'s are grouped to build the grasp stiffness and damping symmetric and p.d. matrices $\mathbf{K}$ and $\mathbf{B}$.

The *Jacobian* $\mathbf{J}$ and *grasp* matrix $\mathbf{G}$ are defined as usual.

Nonlinear dynamics of manipulation and object dynamics are:

$$\mathbf{M}_h\ddot{\mathbf{q}} + \mathbf{Q}_h = -\mathbf{J}^T\mathbf{t} + \tau; \qquad (2)$$

$$\mathbf{M}_o\ddot{\mathbf{u}} + \mathbf{Q}_o = \mathbf{G}\mathbf{t} + \mathbf{w}. \qquad (3)$$

Here, $\mathbf{M}_h$ and $\mathbf{M}_o$ are inertia symmetric and p.d. matrices, while $\mathbf{Q}_h$ and $\mathbf{Q}_o$ are terms including velocity-dependent and gravity forces of the manipulator and of the object, respectively.

Let $\mathbf{q} = \mathbf{q}_o$, $\mathbf{u} = \mathbf{u}_o$, $\dot{\mathbf{q}} = \dot{\mathbf{u}} = \mathbf{0}$, $\tau = \tau_o (= \mathbf{J}^T\mathbf{t}_o)$, $\mathbf{w} = \mathbf{w}_o (= -\mathbf{G}\mathbf{t}_o)$, $\mathbf{t} = \mathbf{t}_o$, be a reference equilibrium configuration, the linear approximation of the dynamics in the neighborhood of such equilibrium point is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau\delta\tau + \mathbf{B}_w\delta\mathbf{w}, \qquad (4)$$

where state, input and disturbance vectors are defined as the departures from the reference equilibrium point:

$$\begin{aligned} \mathbf{x} &= \left[\delta\mathbf{q}^T, \delta\mathbf{u}^T, \delta\dot{\mathbf{q}}^T, \delta\dot{\mathbf{u}}^T\right]^T \\ &= \left[(\mathbf{q}-\mathbf{q}_o)^T\ (\mathbf{u}-\mathbf{u}_o)^T\ \dot{\mathbf{q}}^T\ \dot{\mathbf{u}}^T\right]^T; \\ \delta\tau &= \tau - \mathbf{J}^T\mathbf{t}_o; \\ \delta\mathbf{w} &= \mathbf{w} + \mathbf{G}\mathbf{t}_o \end{aligned} \qquad (5)$$

and the dynamic, input and disturbance matrices $\mathbf{A}$, $\mathbf{B}_\tau$ and $\mathbf{B}_w$ are

$$\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{L}_k & \mathbf{L}_b \end{bmatrix}; \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_h^{-1} \\ \mathbf{0} \end{bmatrix}; \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_o^{-1} \end{bmatrix}, \qquad (6)$$

respectively. To simplify notation we will henceforth omit the symbol δ.

Assuming a locally isotropic model of visco-elastic phenomena, and assuming that gravity and local variations of the Jacobian and grasp matrices are negligible, all the dynamic contributions of terms $\mathbf{Q}_h$ and $\mathbf{Q}_o$ can be neglected and simple expressions are obtained for the blocks $\mathbf{L}_k$ and $\mathbf{L}_b$, as $\mathbf{L}_k = -\mathbf{M}^{-1}\mathbf{P}_k$, and $\mathbf{L}_b = -\mathbf{M}^{-1}\mathbf{P}_b$ where $\mathbf{M} = \mathrm{diag}(\mathbf{M}_h, \mathbf{M}_o)$, and

$$\mathbf{P}_k = \begin{bmatrix} \mathbf{J}^T \\ -\mathbf{G} \end{bmatrix} \mathbf{K} \begin{bmatrix} \mathbf{J} & -\mathbf{G}^T \end{bmatrix};$$

$$\mathbf{P}_b = \begin{bmatrix} \mathbf{J}^T \\ -\mathbf{G} \end{bmatrix} \mathbf{B} \begin{bmatrix} \mathbf{J} & -\mathbf{G}^T \end{bmatrix}.$$

The nullspaces of matrices $\mathbf{J}$ and $\mathbf{G}$ and their transposes have a relevant influence on the dynamic behaviour of the manipulation system. The following grasp properties have been discussed in [9]: A grasp (or manipulation system) is said "defective" if $\ker(\mathbf{J}^T) \neq \mathbf{0}$, "indeterminate" if $\ker(\mathbf{G}^T) \neq \mathbf{0}$, "graspable" if $\ker(\mathbf{G}) \neq \mathbf{0}$ and "redundant" if $\ker(\mathbf{J}) \neq \mathbf{0}$.

The class of grasps with general kinematics this paper deals with consists of configuration systems which result obviously graspable $\ker(\mathbf{G}) \neq \mathbf{0}$ and possibly redundant $\ker(\mathbf{J}) \neq \mathbf{0}$ and kinematically defective $\ker(\mathbf{J}^T) \neq \mathbf{0}$.

3 Controlled outputs: internal forces

The control of contact forces $\mathbf{t}$ is a fundamental part of the manipulation control problem. Contact forces allows the manipulator to maintain the grasp, rejecting external disturbance $\mathbf{w}$ and controlling the object motion. In [9, 10] the reachable subspace of contact forces as outputs of the dynamic system (4) was studied.

Consider the departures of contact force vector $\mathbf{t}$ from the reference equilibrium $\mathbf{t}_o$, define $\mathbf{t}'$ (henceforth $\mathbf{t}$) as its first order approximation which in [9] was computed as an output of the linearized model (4): $\mathbf{t} = \mathbf{C}_t\mathbf{x}$ where $\mathbf{C}_t = [\mathbf{KJ} \;\; -\mathbf{KG}^T \; \mathbf{BJ} \;\; -\mathbf{BG}^T]$; and assume that stiffness matrix $\mathbf{K}$ and damping matrix $\mathbf{B}$ are proportional.

In [3] it was shown that in grasps with general kinematics not all the internal forces are controllable. Thus an analysis of their reachable set is needed in order to specify consistent control goals. In [10] the authors, starting from the linearized dynamics, define the *reachable internal forces subspace* $\mathcal{R}_{ti,\tau}$ as the intersection between the reachable subspace of all the contact forces $\mathcal{R}_{t,\tau}$ and the null space of the grasp matrix:

$$\mathcal{R}_{ti,\tau} = \mathcal{R}_{t,\tau} \cap \ker(\mathbf{G}).$$

Moreover, in terms of column space, they showed that

$$\mathcal{R}_{ti,\tau} = \mathrm{im}(\mathbf{P}_{NG}\, \mathbf{C}_t) = \mathrm{im}(\mathbf{P}_{NG}\, \mathbf{KJ})$$
$$\text{where } \mathbf{P}_{NG} = \mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G}.$$

According to this result, the subspace of reachable internal forces is obtained by the projector $\mathbf{P}_{NG}$ (onto the null space of $\mathbf{G}$) acting on the column space of $\mathbf{C}_t$. By the way, observe that the previous formula states the equality of $\mathcal{R}_{ti,\tau}$ with the active forces in [3] and the asymptotically reachable forces in [10].

Finally, the regulated force output $\mathbf{e}_{ti}$ is chosen as the projection of the force vector $\mathbf{t}$ onto the null space of $\mathbf{G}$, i.e. the reachable internal contact forces:

$$\mathbf{e}_{ti} = \mathbf{E}_{ti}\mathbf{x}; \;\; \text{where } \mathbf{E}_{ti} = \mathbf{P}_{NG}\mathbf{C}_t = [\mathbf{Q}\; \mathbf{0}\; \mathbf{Q}\; \mathbf{0}]$$
$$\text{and } \mathbf{Q} = (\mathbf{I} - \mathbf{KG}^T(\mathbf{GKG}^T)^{-1}\mathbf{G})\mathbf{KJ}. \tag{7}$$

It might be worthwhile to emphasize that

$$\mathcal{R}_{ti,\tau} = \mathrm{im}(\mathbf{E}_{ti}). \tag{8}$$

4 Decoupled output: object motions

It was already pointed out that the control of internal forces, may interact with the object dynamics. It may happen that the internal force control would involve nontrivial transients for the motion of the manipulated object. This problem will be overcame by the noninteracting force/motion control.

In this section we define the rigid-body object kinematics with respect to which the internal force controller will result decoupled.

Rigid-body kinematics are of particular interest in the control of manipulation systems. They do not involve visco-elastic deformations of bodies, thus they can be regarded as low-energy motions. In fact, rigid-body kinematics represent the easiest way to move the object.

Rigid-body kinematics have been studied in a quasi-static setting in [4] and in terms of unobservable subspaces from contact forces in [9]. In both cases rigid kinematics were described by a matrix $\mathbf{\Gamma}$ whose columns form a basis for $\ker[\mathbf{J} \;\; -\mathbf{G}^T] = \mathrm{im}(\mathbf{\Gamma})$ where

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Gamma}_r & \mathbf{\Gamma}_{qc} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}_{uc} & \mathbf{\Gamma}_i \end{bmatrix},$$
$$\text{and } \mathbf{J\Gamma}_{qc} = \mathbf{G}^T\mathbf{\Gamma}_{uc} \tag{9}$$

being $\mathbf{\Gamma}_r$ a basis matrix (b.m.) of the subspace of redundant manipulator motions $\ker(\mathbf{J})$, $\mathbf{\Gamma}_i$ a b.m. of the subspace of indeterminate object motions $\ker(\mathbf{G}^T)$, and $\mathbf{\Gamma}_{qc}$ and $\mathbf{\Gamma}_{uc}$ conformal partitions of a complementary basis matrix[1].

The column space of $\mathbf{\Gamma_c} = \begin{bmatrix} \mathbf{\Gamma}_{qc} \\ \mathbf{\Gamma}_{uc} \end{bmatrix}$ consists of coordinated rigid-body motions of the mechanism, for the manipulator ($\mathbf{\Gamma}_{qc}$) and the object ($\mathbf{\Gamma}_{uc}$) components. Physically rigid-body displacements do not involve variation of contact forces.

In [9], it has been shown that rigid-body motions are reachable, i.e. they belong to the space of reachability of linear system (4) with input the vector of joint generalized forces τ. Note that the rigid-body subspace is only a subspace of the reproducible one which also contains motions due to deformations of elastic elements in the model.

The object-position regulated output $\mathbf{e}_{uc}$ is chosen as the projection, through $\mathbf{\Gamma}^P_{uc}$, of object positions $\mathbf{u}$ onto the subspace of rigid-body object motions $\mathrm{im}(\mathbf{\Gamma}_{uc})$:

$$\mathbf{e}_{uc} = \mathbf{E}_{uc}\mathbf{x}; \;\; \text{where } \mathbf{E}_{uc} = \mathbf{\Gamma}^P_{uc}[\mathbf{0}\; \mathbf{I}\; \mathbf{0}\; \mathbf{0}]$$
$$\text{and } \mathbf{\Gamma}^P_{uc} = \mathbf{\Gamma}_{uc}(\mathbf{\Gamma}^T_{uc}\mathbf{\Gamma}_{uc})^{-1}\mathbf{\Gamma}^T_{uc}. \tag{10}$$

5 Internal force control example

The following example shows the behaviour of an internal force controller designed for rigid-body systems but acting on a manipulation system with a deformable object. The planar manipulation system is the two Dof's depicted in fig. 1-a). It has contact points in[2] $(0,2)$ and $(2,2)$, the prismatic joint in $(2,1)$, and stiffness, damping and inertia matrices normalized to the identity matrix. The Jacobian and the grasp matrix take the following values

$$\mathbf{J} = \begin{bmatrix} -2 & 0 \\ 0 & 0 \\ -2 & -1 \\ 2 & 0 \end{bmatrix}; \; \mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix}.$$

Observe that the system is defective, cf. [10], $\ker(\mathbf{J}^T) \neq \emptyset$. The controlled output is $\mathbf{e}_{ti}$ (7): the projection of the

[1] $\mathbf{W}$ is called a complementary basis matrix of $\mathcal{V}$ to $\mathcal{X}$ if it is f.c.r. and $\mathrm{im}(\mathbf{W}) \oplus \mathcal{V} = \mathcal{X}$.

[2] With respect to the depicted reference frame.

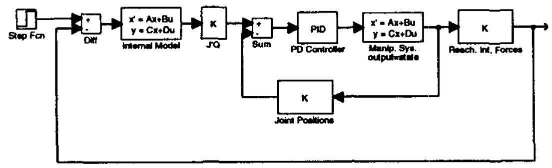

Figure 3: Two-loops internal force control law.

force vector $\mathbf{t}$ onto the null space of $\mathbf{G}$, i.e. the subspace of reachable internal forces which, in the case study, is $\mathcal{R}_{ti,\tau} = \text{im}([1\ 0\ -1\ 0])$.

The force control block diagram is reported in fig. 3. It consists of two loops: the internal one is the stabilizing (cf. [9]) PD controller of joint positions while the external one is the force loop composed by the internal model and the Jacobian transpose.

The internal model principle guarantees that when the input is a step, the magnitude of the output $\mathbf{e}_{ti}$ reaches asymptotically the step value. Unfortunately the controller action moves the grasped object along the rigid body coordinate subspace, $\text{im}(\Gamma_{uc}) = \text{im}\,[-0.76\ 0.38\ 0.38]$. Simulation results are reported in fig. 4 and 5. The final configuration of the manipulation system looks like the one depicted in fig. 1-b). Obviously this is a dangerous maneuver in applications where high precision is required. The paper contribution consists in proposing a feedback internal force controller whose action results to be decoupled from the rigid-body object dynamics.

6 Object-decoupling control of internal forces

This section is aimed at the analysis of the object-motion decoupling control of internal forces for general grasping mechanisms.

Definition 1 *Consider the dynamic system (4). A control law of the internal forces* $\mathbf{e}_{ti}$ *is* **decoupled** *with respect to the coordinate rigid-body motions* $\mathbf{e}_{uc}$*, if there exists a linear combination* $\mathbf{B}_\tau \mathbf{U}$ *of the input-matrix columns such that, for zero initial condition, the input* $\tau = \mathbf{U}\tau_n$ *only affects the internal forces* $\mathbf{e}_{ti}$ *while* $\mathbf{e}_{uc}$ *remains identically zero.*

The following theorem shows that a state feedback gain exists such that, the closed-loop system is decoupled in the sense of Definition 1.

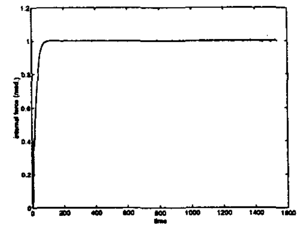

Figure 4: Simulation result of block diagram 3, reachable internal force.

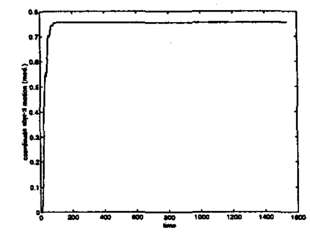

Figure 5: Simulation result of block diagram 3, coordinate rigid-body object positions.

Theorem 1 *Consider the linearized manipulation system (4). Under the hypothesis that the grasp system is not indeterminate,* $\ker(\mathbf{G}^T) = \mathbf{0}$*, there exists a stabilizing state feedback* $\mathbf{F}$ *and a matrix* $\mathbf{U}$ *such that*

- *the decoupling condition of Definition 1 holds*
- $\text{im}\,(\mathbf{E}_{ti} < \mathbf{A} + \mathbf{B}_\tau \mathbf{F} \mid \mathbf{B}_\tau >) = \text{im}(\mathbf{E}_{ti})$.

being $< \mathbf{A} \mid \mathbf{B} >= [\mathbf{B}\ \mathbf{AB}\ \mathbf{A}^2\mathbf{B} \cdots \mathbf{A}^{n-1}\mathbf{B}]$ *where* $\mathbf{A} \in \mathbb{R}^{n \times n}$.

Theorem 1 states both the decoupling property and the complete output reachability of internal forces $\mathcal{R}_{ti,\tau}$ (8). The control scheme implementing the internal force decoupling control is that reported in fig. 6 (framed).

If the sensor system of the robotic manipulator is not able to measure the object position and velocity, a state observer is needed. This commonly occurs when robots are equipped with tactile sensors. If contact force and joint position sensors are chosen, the local state detectability from the measured outputs has been proved [9].

Since the generality of this theorem, we can state that the force/motion decoupling property can be considered a structural property of grasps with general kinematics.

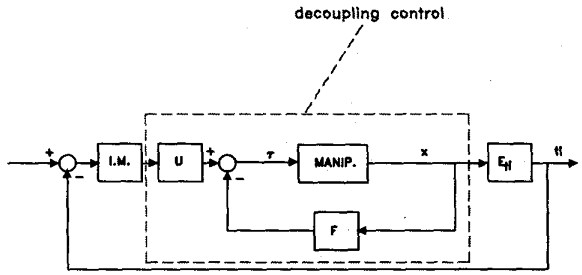

Figure 6: Internal state-feedback decoupling controller and external loop for the internal force control (I.M. is the internal model of the reference signal).

7 Controller design

The complete proof of the decoupling theorem is reported in [8]. The groundwork of the decoupling theorem is the geometric approach to the multivariable control design, see [1] and therein references. Next, we recall a basic concept of this approach. Consider the linear dynamic system $(\mathbf{A}, \mathbf{B}_\tau)$ in (4) and a given subspace of the state space, say $\mathcal{S}$. Introduce the notion of *$\mathcal{S}$-constrained reachable subspace $\mathcal{R}_\mathcal{S}$*, as the reachability subspace with the constraint that state trajectories belong to $\mathcal{S}$. The subspace of constrained state reachability $\mathcal{R}_\mathcal{S}$ is given by, [1]

$$\mathcal{R}_\mathcal{S} = \mathrm{im} < \mathbf{A} + \mathbf{B}_\tau \mathbf{F} \mid \mathbf{B}_\tau \mathbf{U} > \subseteq \mathcal{S} \qquad (11)$$

$$\text{with } \mathbf{F}: \quad (\mathbf{A} + \mathbf{B}_\tau \mathbf{F})\mathcal{S} \subseteq \mathcal{S}, \qquad (12)$$

$$\mathbf{U}: \quad \mathrm{im}(\mathbf{B}_\tau \mathbf{U}) = \mathrm{im}(\mathbf{B}_\tau) \cap \mathcal{S}. \qquad (13)$$

According to the theorem statement, it's now clear that to decouple the system, this should be controlled by constraining state trajectories to lie onto the null space of the rigid-body output matrix $\mathbf{E}_{uc}$ (10).

The state-feedback control law is reported in fig. 6. It consists of a stabilizing feedback gain $\mathbf{F}$ which satisfies inclusion (12), where $\mathcal{S} := \max \mathcal{V}(\mathbf{A}, \mathbf{B}, \ker(\mathbf{E}_{uc}))$ is the maximal controlled invariant contained in $\ker(\mathbf{E}_{uc})$ and of the input selection matrix $\mathbf{U}$ defined by equation (13).

Note that maximal controlled invariant $\mathcal{S}$ is computed as the limit of a recursive geometric algorithm converging in a finite number of steps [1].

7.1 Algebraic output decoupling of internal forces

In previous section, decoupling control is obtained by feeding back the state vector. In some cases, decoupling control of internal forces can be obtained by means of an algebraic output feedback control, cf. [2], from the sensed output. Let the measurement vector $\mathbf{y}_m$ consist of contact forces $\mathbf{t}$ and manipulator joint positions $\mathbf{q}$:

$$\mathbf{y}_m = \mathbf{C}_m \mathbf{x}$$
$$\mathbf{C}_m = \begin{bmatrix} \mathbf{I}_{q\times q} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{KJ} & -\mathbf{KG}^T & \mathbf{BJ} & -\mathbf{BG}^T \end{bmatrix}. \qquad (14)$$

The following theorem, proven in [8], holds

Theorem 2 *Consider the linearized manipulation system (4). Under hypotheses that* $\ker(\mathbf{G}^T) = \mathbf{0}$ *and that* $\ker(\Gamma_{uc}^T)$ *is* $\mathbf{M}_o^{-1}\mathbf{GKG}^T$*-invariant, there exists an algebraic output feedback* $\tau = \mathbf{L}\mathbf{y}_m$ *and a matrix* $\mathbf{U}$ *such that*

- *the decoupling condition of Definition 1 holds and*
- $\mathrm{im}\,(\mathbf{E}_{ti} < \mathbf{A} + \mathbf{B}_\tau \mathbf{L} \mathbf{C}_m \mid \mathbf{B}_\tau \mathbf{U} >) = \mathrm{im}(\mathbf{E}_{ti})$.

8 Decoupled internal force control

The case study has been introduced in Section 5 together with the non-decoupling control law reported in fig. 3. Simulation results of Section 5 are here compared with those obtained by using the decoupling controller of Theorem 1.

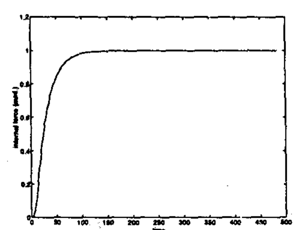

Figure 7: Simulation result of block diagram 6, reachable internal force.

The state-feedback gain $\mathbf{F}$ and the input selection matrix $\mathbf{U}$ of Theorem 1 are computed according to Section 7 as

$$\mathbf{F} = \begin{bmatrix} 5 & 2 & 4 & -4.5 & -4.5 & 4 & 2 & 4 & -4.5 & -4.5 \\ 2 & -6 & 11 & 10 & 10 & 2 & -7 & 11 & 10 & 10 \end{bmatrix}$$

and

$$\mathbf{U} = \begin{bmatrix} -0.16 \\ 0.99 \end{bmatrix}.$$

The controlled output is $\mathbf{e}_{ti}$: the projection of the force vector $\mathbf{t}$ onto the null space of $\mathbf{G}$, i.e. the reachable internal forces

$$\mathcal{R}_{ti,\tau} = \mathrm{im}([1\ 0\ -1\ 0]).$$

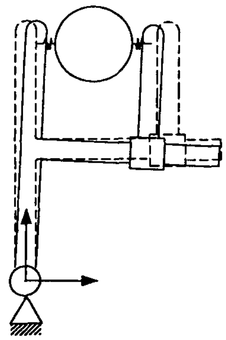

Figure 8: Decoupling internal force control.

Again the internal model principle guarantees that, with a unit step as input, the internal force e_{ti} reaches (in magnitude) the unit, but unlike the controller of fig. 6, the decoupling control of fig. 6 does not affect the rigid-body object motion which remains identically zero. In other words, the proposed internal force controller allows one to squeeze the object, see fig. 7, in a way that the manipulated object does not change its position.

Fig. 8 pictorially describes the squeezing action by the decoupling controller.

9 Conclusions

In this paper we considered the problem of controlling internal forces of grasps with general kinematics.

Special attention was devoted to manipulation systems with significant compliance at the contacts. As a consequence, the object dynamics was taken into account in controlling the internal forces.

After characterizing the rigid-body object motions and the reachable contact forces, we focused on the problem of synthesizing an internal force control law which does not interact with the rigid-body object motion.

The geometric approach is used throughout the paper whose main result states that there always exists a feedback control law that decouples the internal force control action from the object dynamics. Note that such a decoupling property can be considered as a structural property of manipulation systems with general kinematics.

The linearization method employed in this paper is a reasonable approach for internal force control since it does not involve large changes of the grasp configuration. Future research involves the generalization of this result to the full nonlinear model.

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