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Supervisory Switching Control in Robotic Manipulation

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Abstract

A switching controller for a class of robotic manipulators with grasping capabilities is presented. The aim is to control the motion of the grasped object along a desired trajectory while complying with contact force constraints.

1 Introduction

This paper deals with the position/force control in cooperative robotic manipulation of an object whereby, the motion task of the system, can be divided into two different subtasks, i.e. tracking a desired object trajectory, while fulfilling a set of constraints on the contact forces applied to the object. A possible approach, to effectively attack problems arising with the difficulty of modeling the system in all the possible configurations robot–object, is to use several different controllers and to switch among them with a logical device according to some performance criterion. If this logic unit, called supervisor, orchestrates the switching between different controllers depending on some input–output observation of the system, then the control algorithm is intrinsically adaptive. Such approach is distinct from the gain–scheduling paradigms, e.g. [12], which are not based on some amplitude performance criterion and compute continuous controls even when the design is based on a collection of controllers.

This paper combines a logic-based switching control methodology extensively studied in [6, 7, 2, 4], with a decoupling technique to control position/force trajectories in robotic manipulation [10]. The logic–based switching technique has already been applied to robotic problems. For instance, in [1] authors emphasized the effectiveness of switched control systems with respect to stabilization and performance for redundant manipulators. The idea of using switching control techniques in the adaptive control context to improve tracking performance has been recently emphasized in [8], where switching control is used to select among a set of free parallel running adaptive algorithms.

The present work is based on [10] where authors focus on one of the central problems of robotic manipulation, i.e., controlling the manipulator in order to track a desired object trajectory, while fulfilling contact constraints (friction bounds, etc.) at every instant. The main result consists in the suggestion of the organization of the output object–position/contact–force vector, which results functional controllable, exhausts the control capabilities and incorporates the constraints as well as the task requirements for the manipulation system. Such result applies to a wide class of manipulation systems and can be considered as a structural property. Unfortunately, in spite of its generality, the nature of this result is local as it is based on the linearization of the mechanical system dynamics.

The aim of this paper consists in generalizing the results of [10] to the full nonlinear model by means of the logic-based switching technique. Here, the way to track the position/force reference trajectory is based on the selection in real time of an "appropriate controller" between a set of prespecified off-the-shelf controllers, built on a set of ad hoc linearized models of the mechanical system. Such controllers are selected according to a model selection rule based on minimization of a performance index, [6]. According to this rule, an "appropriate controller" is placed into the control loop from time to time, in such a way that the output prediction error between the real system output and the output of the model, upon the controller is designed, is minimized. This choice strategy interprets the concept of certainty equivalence [4, 5], and ensures that the controlled system always incorporates a control law based on the model which best reproduces the system behaviour along its trajectory.

2 General manipulation systems and force/position decoupling

The class of "general manipulation systems" this paper is concerned with is comprised of mechanisms with any number of limbs (open kinematic chains), of joints (prismatic, rotoidal, spherical, etc.) and of contacts between a reference member called "object" and links in any position of the limb chains. This class includes co–operating robots, industrial grippers, robotics hands and so forth, cf. [11, 10] and references therein. As a paradigm for general manipulation systems, we refer to the case of a multifingered hand manipulating an object through contacts on its finger parts.

Regarding notation, let $\mathbf{q} \in \mathbb{R}^q$ be the vector of joint positions, $\tau \in \mathbb{R}^q$ the vector of joint forces and/or torques, $\mathbf{u} \in \mathbb{R}^d$ the vector locally describing the position and the orientation of a frame attached to the object and $\mathbf{w} \in \mathbb{R}^d$ the vector of external disturbances acting on the object. Finally let us introduce the vector $\mathbf{t} \in \mathbb{R}^t$ including forces and torques at all contacts between the robotic device and the manipulated object.

Assume that contact forces arise from a lumped-parameter model of visco-elastic phenomena at the contacts, summarized by the stiffness matrix $\mathbf{K}$ and the damping matrix $\mathbf{B}$. The Jacobian $\mathbf{J}$ and the grasp matrix $\mathbf{G}$ are defined as usual as the linear maps relating the velocities of the contact points on the links and on the object, to the joint and object velocities, respectively. For a complete description of the model, the reader is referred to [10, 11].

The nonlinear dynamics of a general manipulation systems is obtained as

$$\begin{array}{rcl} \ddot{\mathbf{q}} & = & \mathbf{M}_h^{-1}(-\mathbf{Q}_h - \mathbf{J}^T\mathbf{t} + \tau); \\ \ddot{\mathbf{u}} & = & \mathbf{M}_o^{-1}(-\mathbf{Q}_o + \mathbf{G}\mathbf{t} + \mathbf{w}); \\ \mathbf{t} & = & \mathbf{K}\mathbf{J}(\mathbf{c}_m - \mathbf{c}_o) + \mathbf{B}\delta(\dot{\mathbf{c}}_m - \dot{\mathbf{c}}_o). \end{array} \quad (1)$$

where $\mathbf{M}_h$ ($\mathbf{M}_o$) is the symmetric and positive definite inertia matrix of the hand (object); $\mathbf{Q}_h$ ($\mathbf{Q}_o$) is the term including velocity-dependent and gravity forces of the hand (object) and $\mathbf{c}_m$ ($\mathbf{c}_o$) is the vector of contact points thought as attached to the hand (object).

In [11, 10], the manipulation system dynamics is linearized at a reference equilibrium configuration

$$\{\tau, [\mathbf{q}, \mathbf{u}, \dot{\mathbf{q}}, \dot{\mathbf{u}}], \mathbf{t}\} = \{\tau_o, [\mathbf{q}_o, \mathbf{u}_o, \mathbf{0}, \mathbf{0}], \mathbf{t}_o\} \quad (2)$$

and in the neighborhood of such an equilibrium the linearized dynamics of the manipulation system is written as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau\tau' + \mathbf{B}_w\mathbf{w}', \quad (3)$$

where state $(\mathbf{x} = [(\mathbf{q}-\mathbf{q}_o)^T \ (\mathbf{u}-\mathbf{u}_o)^T \ \dot{\mathbf{q}}^T \ \dot{\mathbf{u}}^T]^T)$, input $(\tau' = \tau - \mathbf{J}^T\mathbf{t}_o)$ and disturbance vectors $(\mathbf{w}' = \mathbf{w} + \mathbf{G}\mathbf{t}_o)$ are defined as the departures from the equilibrium configuration. In [10], it has been shown that a simple PD control of joints variables, asymptotically stabilizes the system in a neighborhood of the equilibrium point. Henceforth the manipulation system will be considered with the PD control and the resulting dynamic matrix of the linearized mode will be indicated as $\mathbf{A}_f$. Note that the input τ' to feedback system $(\mathbf{A}_f, \mathbf{B}_\tau)$ corresponds to the change of the joint references with respect to their equilibria.

2.1 Position/force control and decoupling

One of the main goals of robotic manipulation control is to follow a given trajectory with the manipulated object while guaranteeing that contact forces comply with contact constraints thus ensuring the grasp stability [9, 10]. In the following the position/force controlled outputs will be introduced.

With reference to object trajectories, rigid-body kinematics play a key role in manipulation control: they do not involve visco-elastic deformations at contacts and can be regarded as low-energy motions. In [11] rigid kinematics were described by the basis matrix Γ whose columns form a basis for $\ker[\mathbf{J} \ -\mathbf{G}^T] = \mathrm{im}(\Gamma) = \mathrm{im}[\Gamma_{qc}^T \ \Gamma_{uc}^T]^T$ where Γ_{uc} (Γ_{qc}) is a basis matrix of the coordinated rigid-body motions of the object (manipulator) part. Observe that to simplify notation, it is assumed here that $\ker(\mathbf{J}) = \{\mathbf{0}\}$ and that $\ker(\mathbf{G}^T) = \{\mathbf{0}\}$.

The controlled output vector of object positions is interpreted by the *rigid-body* object motion $\mathbf{u}_c$ defined as the projection of the object displacement $\mathbf{u}$ onto the column space of Γ_{uc}:

$$\begin{array}{l} \mathbf{u}_c = \mathbf{E}_{uc}\mathbf{x}; \quad \text{where} \\ \mathbf{E}_{uc} = \Gamma_{uc}^T[\mathbf{0} \ \mathbf{I} \ \mathbf{0} \ \mathbf{0}]. \end{array} \quad (4)$$

The second output vector to be controlled consists of internal contact forces. These are self-balanced forces, belong to the null space of the grasp matrix $\mathbf{G}$ and enable the robotic device to grasp the object. In general, not all the internal forces are reachable by control inputs thus we choose as second controlled output the *reachable internal* contact forces $\mathbf{t}_i$ defined as the projection of the force vector $\mathbf{t}$ onto the null space of $\mathbf{G}$:

$$\begin{array}{l} \mathbf{t}_i = \mathbf{E}_{ti}\mathbf{x}; \quad \text{where} \\ \mathbf{E}_{ti} = \mathbf{Q}^T[\mathbf{Q} \ \mathbf{0} \ \mathbf{Q} \ \mathbf{0}] \quad \text{and} \\ \mathbf{Q} = (\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J} \end{array} \quad (5)$$

The following theorem proven in [10], shows that the task-oriented controlled output vector

$$\mathbf{e} = \begin{bmatrix} \mathbf{u}_c \\ \mathbf{t}_i \end{bmatrix} = \mathbf{E}\mathbf{x}; \quad \text{with } \mathbf{E} = \begin{bmatrix} \mathbf{E}_{uc} \\ \mathbf{E}_{ti} \end{bmatrix} \quad (6)$$

exhausts the control capabilities (square system) and is functionally controllable.

Theorem 1 *Under the hypothesis that* $\ker(\mathbf{G}^T) = \mathbf{0}$ *and that* $\ker(\mathbf{J}) = \mathbf{0}$*, the linearized dynamics in Section 2 described by the triple* $(\mathbf{A}_f, \mathbf{B}_\tau, \mathbf{E})$ *is asymptotically stable, square* $(\tau', \mathbf{e} \in \mathbb{R}^q)$ *and at* $s = 0$ *det*$(\mathbf{E}(s\mathbf{I} - \mathbf{A}_f)^{-1}\mathbf{B}_\tau) \neq 0$*. This implies the asymptotic reproducibility and functional controllability [3].*

It should be remarked that the first hypothesis in Theorem 1 is structural while the second is technical [10]. The asymptotic reproducibility property of dynamics means that it is possible to asymptotically decouple and track force and position step references by applying the control input

$$\tau = -(\mathbf{E}\mathbf{A}_f^{-1}\mathbf{B}_\tau)^{-1}\begin{bmatrix} \mathbf{u}_{c,ref} \\ \mathbf{t}_{i,ref} \end{bmatrix}. \quad (7)$$

Unfortunately, the open-loop decoupling control (7) is based on the approximate linearized model. Then it applies only locally around the equilibrium points.

In the rest of the paper an effort is made to recast the problem of position/force control in a logic–based switching framework. The aim is to extend Theorem 1 to the full nonlinear model (1) of general manipulation systems.

3 Logic–based switching framework

The switching technique is based on a multiestimator stage having as inputs the joint torques $\tau \in \mathbb{R}^q$ and the measured outputs $\mathbf{y} \in \mathbb{R}^m$ of the robotic nonlinear MIMO system. Assume that the sensed outputs of the nonlinear system are the joint positions and the contact forces in (1): $\mathbf{y} = [\mathbf{q}^T, \mathbf{t}^T]^T$. Note that controlled output $\mathbf{e}$ in (6), differs from the informative output $\mathbf{y}$.

Nonlinear dynamics (1), measurements $\mathbf{y}$ and position/force controlled output $\mathbf{e}$ equations of robotic manipulation systems are rewritten as

$$\begin{aligned} \dot{\mathbf{x}} &= A_M(\mathbf{x}, \tau, \mathbf{w}); \\ \mathbf{y} &= C_M(\mathbf{x}); \\ \mathbf{e} &= \begin{bmatrix} \mathbf{u}_c \\ \mathbf{t}_i \end{bmatrix} \end{aligned} \qquad (8)$$

where the state $\mathbf{x} = [\mathbf{q}^T, \dot{\mathbf{q}}^T, \mathbf{u}^T, \dot{\mathbf{u}}^T]^T \in \mathbb{R}^n$ is finite dimensional and $\mathbf{w}$ is the input disturbance.

The *control goal* consists of asymptotically tracking a position/force reference signal $\mathbf{r} = [\mathbf{u}_{c,ref}^T, \mathbf{t}_{i,ref}^T]^T$ by means of a logic–based switching multiestimator and controller.

3.1 Equilibrium points and linearized models

With no loss of generality, from now on assume that $\mathbf{w} = 0$ in (1) and (8). Consider a number N_{eq} of linearized model $(\mathbf{A}_{f,p}, \mathbf{B}_{\tau,p})$, cf. Section 2, built around N_{eq} equilibrium points, as in (2), identified with triples $\{\tilde{\tau}_p, \tilde{\mathbf{x}}_p, \tilde{\mathbf{y}}_p\}$ $(p = 1, \ldots N_{eq})$ such that

$$A_M(\tilde{\mathbf{x}}_p, \tilde{\tau}_p) = 0; \quad \tilde{\mathbf{y}}_p = C_M(\tilde{\mathbf{x}}_p); \quad \tilde{\mathbf{e}} = [\tilde{\mathbf{u}}_c^T, \tilde{\mathbf{t}}_i^T]^T. \quad (9)$$

For small perturbations around the equilibrium point $\tilde{\mathbf{x}}_p$ the dynamic behaviour of the nonlinear system is modeled as discussed in Section 2 and in [10]:

$$\delta\dot{\mathbf{x}}_p = \mathbf{A}_{f,p}\delta\mathbf{x}_p + \mathbf{B}_{\tau,p}\delta\tau_p, \quad \delta\mathbf{y}_p = \mathbf{C}_p\delta\mathbf{x}_p; \quad \delta\mathbf{e}_p = \mathbf{E}_p\delta\mathbf{x}_p;$$

where $\mathbf{C}_p$ is the jacobian of C_M in (8) evaluated at the equilibrium point $\mathbf{x}_p$. Henceforth symbol δ will be omitted.

Assumption 1 *For control purposes, the behaviour of the control input τ versus the measured output $\mathbf{y}$ of the system in (8) is described with a model $\mathcal{M}$, whereby $\mathcal{M}$ is an unknown member of a suitably defined family $\mathcal{F}_M$ of the type*

$$\mathcal{F}_M = \bigcup_{p \in \mathcal{P}} \mathcal{M}_p(1 + \delta\mathcal{M}_p) \qquad (10)$$

where $\mathcal{M}_p$ are $m \times q$ transfer function matrices indexed by a parameter p belonging to a finite set $\mathcal{P} = \{1, \ldots, N_{eq}\}$. Each family of models of the system is centered about a nominal transfer function matrix $\mathcal{M}_p$, and includes unmodeled perturbation $\delta\mathcal{M}_p$, that in our case takes into account for the system (8) intrinsic nonlinearity. We assume also that $\delta\mathcal{M}_p$ is 'small' enough in such a way that for each possible system $\mathcal{M}$ in $\mathcal{F}_M$, there would always exists a nominal system model $\mathcal{M}_p$ within the set of models transfer matrices which is 'close' to $\mathcal{M}$ in some suitably defined sense.

The reason why a description of the type in (10) has been introduced in this context is as follows. We shall mean by using a model like $\mathcal{F}_M$, that all the possible dynamic configurations of the robotic manipulation system during its task execution can be 'covered' with an ad hoc chosen set of models $\mathcal{M}_p = \mathbf{C}_p(s\mathbf{I} - \mathbf{A}_{f,p})^{-1}\mathbf{B}_{\tau,p}$, linearized around N_{eq} equilibrium points, plus some relative perturbation $\delta\mathcal{M}_p$ taking into account for the intrinsic unmodeled system nonlinearity.

3.2 Closed loop decoupling multicontrollers

For each nominal model member $\mathcal{M}_p = \mathbf{C}_p(s\mathbf{I} - \mathbf{A}_{f,p})^{-1}\mathbf{B}_{\tau,p}$ of $\mathcal{F}_M$, we have a controller $\mathcal{C}_p$, indexed by $p \in \mathcal{P}$ belonging to a family $\mathcal{C}$ of previously designed controllers with the requirement that the feedback interconnection of $\mathcal{M}_p$ with $\mathcal{C}_p$ is asymptotically stable and asymptotically reproducible, i.e. asymptotically decoupled.

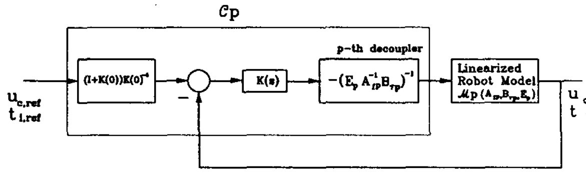


Figure 1: Asymptotic reproducible controller $\mathcal{C}_p$ for linearized model $\mathcal{M}_p$.

The controller $\mathcal{C}_p$ is described in fig. 1 and consists of
- the p–th open–loop decoupler in (7) which asymptotically decouples the position $\mathbf{u}_c$ and the internal force $\mathbf{t}_i$ outputs of the p–th linearized model $\mathcal{M}_p$,
- a linear controller $K(s)$, with invertible finite gain $K(0)$, stabilizing the closed loop for any linearized dynamics $\mathcal{M}_p$, $p \in \mathcal{P}$,
- a pre–compensator $(I + K(0))K(0)^{-1}$ able to compensate the steady state position/force closed–loop error arising when $K(s)$ does not have poles in zero.

For a fixed p, the closed loop controller in fig. 1 can track step inputs for $\mathbf{u}_c$ and $\mathbf{t}_i$ only when the robotic device is 'close' to the equilibrium state $\mathbf{x}_p$. When this is not the case, according to the approach followed in [6, 4], we propose a switching control system of the type depicted in fig. 2. It consists of the feedback interconnection of the *robotic system* $\mathcal{M}$, a *multicontroller* structure C_σ and a *linear multiestimator-based supervisory switching logic* $\mathcal{S}$. Note that the multiestimator

($\mathcal{LEM}_p$) behaviour is affected by the sensed output $\mathbf{y} = [\mathbf{q}^T, \mathbf{t}^T]^T$ and not by the controlled one.

The multicontroller structure will be commanded in such a way that, each controllers of the family will be selected depending on the value of a switching signal σ ($\sigma \in \mathcal{P}$) generated by the logic $\mathcal{S}$.

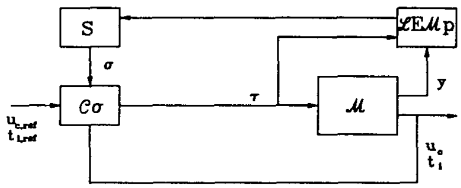


Figure 2: Switching controlled system and robotic manipulation system $\mathcal{M}$.

4 Supervisor based on MIMO linear multiestimator

In this section the linear multiestimator system is derived from that described in [6]. Let assume to be given the nominal model $m \times q$ transfer matrix $\mathcal{M}_p$, of the p–th linearized model of $\mathcal{M}$:

$$\mathcal{M}_p(s) = \begin{bmatrix} \frac{\beta_p^{11}(s)}{\alpha_p^1(s)} & \cdots & \frac{\beta_p^{1q}(s)}{\alpha_p^1(s)} \\ \vdots & \cdots & \vdots \\ \frac{\beta_p^{m1}(s)}{\alpha_p^m(s)} & \cdots & \frac{\beta_p^{mq}(s)}{\alpha_p^m(s)} \end{bmatrix} \quad (11)$$

where the monic polynomials $\alpha_p^i(s)$ and $\beta_p^{ij}(s)$, ($i = 1, \ldots, m, j = 1, \ldots, q, p \in \mathcal{P}$) are the polynomials corresponding respectively, to the denominators and numerators of each row of $\mathcal{M}_p$, and s is the complex variable. The following *linear multiestimator* $\mathcal{LEM}_p$ of $\mathcal{M}_p$ is defined as ($i = 1 \ldots m$, ˙ $p \in \mathcal{P}$)

$$\begin{aligned} \dot{x}_{E,i} &= \tilde{A}_E x_{E,i} + \tilde{G}_E y_i + \tilde{B}_E \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_q \end{bmatrix} && (12) \\ y_{p,i} &= \left[-h^i_{\alpha_p}, h^{i1}_{\beta_p}, \cdots, h^{iq}_{\beta_p} \right] x_{E,i} = c_{p,i}\, x_{E,i} && (13) \\ e_{p,i} &= y_{p,i} - y_i && (14) \end{aligned}$$

where

$$\tilde{A}_E = \begin{bmatrix} A_E & 0 & \cdots & \cdots & 0 \\ 0 & A_E & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & A_E \end{bmatrix}_{\bar{q} \times \bar{q}}, \quad \tilde{G}_E = \begin{bmatrix} b_E \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{\bar{q} \times 1},$$

$$\tilde{B}_E = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ b_E & 0 & \cdots & 0 \\ 0 & \ddots & 0 & \vdots \\ \vdots & \ddots & b_E & 0 \\ 0 & \cdots & 0 & b_E \end{bmatrix}_{\bar{q} \times q}.$$

The above equations (12)-(14) represent a set of estimators for the ith component y_i of the sensed robot output $\mathbf{y} = [\mathbf{q}^T, \mathbf{t}^T]^T \in \mathbb{R}^m$. For each fixed $i \in \{1, \ldots, m\}$ all the estimators share the state $x_{E,i}$. In particular, in (12)-(14): p is the model index; $\bar{q} = q+1$; $i = 1, 2, \ldots, m$ is the row index of $\mathcal{M}_p$; $x_{E,i}$ is a shared state of dimensions $n\bar{q}$, where n is an upper bound on the McMillan degree of all the transfer functions element of $\mathcal{M}_p$, that is, $n > \max_{i,p}\{\deg(\alpha_p^i(s))\}$, $i = 1, \ldots, m$, $p \in \mathcal{P}$; $e_{p,i}$ is the ith component of the output prediction error; A_E is a square $n \times n$ stability matrix such that for any of its eigenvalues λ, it holds that $Re\lambda \leq -\lambda_E < 0$; the couple (A_E, b_E) is controllable, and finally vectors $h^i_{\alpha_p}, h^{ij}_{\beta_p}$, $i = 1, \ldots, m$, $j = 1, \ldots, q$ composing $c_{p,i}$, are obtained from the matrix transfer functions $\mathcal{M}_p$ (11) in such a way that,

$$\left\{ \tilde{A}_E + \tilde{G}_E c_{p,i},\ \ \tilde{B}_E,\ \ c_{p,i} \right\} \approx \left[\frac{\beta_p^{i1}(s)}{\alpha_p^i(s)}, \cdots, \frac{\beta_p^{iq}(s)}{\alpha_p^i(s)} \right]$$

where by symbol $\approx$ we mean "is a realization of".

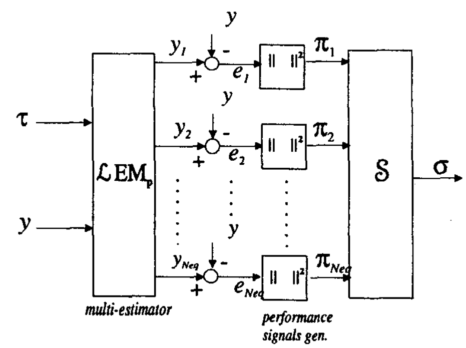


Figure 3: Multiestimator based supervisor.

The *supervision task* of switching among different controllers is managed by the cascade interconnection of the three subsystems in fig. 3: the *linear multiestimator* $\mathcal{LEM}_p$ (12)–(14), a *performance signal generator* Π, and a *dwell time switching logic* $\mathcal{S}$.

The *linear multiestimator* produces as outputs a set of local output prediction errors $e_p = y_p - y$, with $e_p \in \mathbb{R}^m$, and $p \in \mathcal{P}$, being $\mathbf{y}$ the measured outputs of the real system and $\mathbf{y}_p$ those of the linearized model $\mathcal{M}_p$. The following remark holds.

Remark 1 *The same properties stated for the linear multiestimators presented for example in [6], holds also* locally *for the above linear estimator. In particular, under the feedback interconnection $y = y_p$, $\mathcal{LEM}_p$ would have to exhibit the same input-output behaviour between τ and y_p as $\mathcal{M}_p$ does between its input and output. This is true since it can be easily proved that, when $y = y_p$, (12),(13) represents an asymptotic realization of the $\mathcal{M}_p$. Therefore, when $\mathcal{M}$ tends to $\mathcal{M}_p$, e_p tends to zero as $e^{-\lambda_E t}$.*

The *performance generator* is a dynamic system with inputs e_p's and outputs the performance signals π_p's for each linearized model $\mathcal{M}_p$. Signal π_p is evaluated by integrating over time a measure of the distance between the input–output behaviour of the linearized model $\mathcal{M}_p$, and the input–output behaviour of the actual robotic system. Namely, π_p is computed as

$$\dot{\pi}_p(t) = -\lambda\pi_p(t) + \sum_{i=1}^{q} e_{p,i}^2(t), \quad p \in \mathcal{P}, \quad 0 < \lambda < \lambda_E. \quad (15)$$

The *dwell time switching logic* $\mathcal{S}$ discussed in [6] has been used. It has the role of selecting the controller index σ on the basis of the performance signals π_p of each linearized model by choosing from time to time σ equal to that value of p for which π_p is the smallest. The minimum amount of time that is allowed to elapse between two successive controller switchings is regulated by –that means coincides with– the dwell time τ_D. The dwell time is introduced in order to let the stable dynamics of the closed loop switched system have enough time to decay before the next switching occurs. In this way it is prevented that the norm of the composite transition matrix eventually grows without bounds.

5 Simulation results

An application of the logic–based switching controller to the simple planar (2D) manipulation system of fig. 4, is reported. The system has 2 joints and

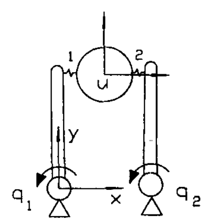


Figure 4: Simple 2–joints, 2–contacts 2D general manipulation system. In this configuration, $q_1 = q_2 = 0$, $\mathbf{u} = [\mathbf{u}_x, \mathbf{u}_y, \theta]^T = [1.5, 3, 0]^T$.

2 contact points, $\mathbf{q} = [q_1, q_2]^T \in \mathbb{R}^2$, $\tau \in \mathbb{R}^2$, $\mathbf{u} = [u_x, u_y, u_\theta]^T \in \mathbb{R}^3$, $\mathbf{t} \in \mathbb{R}^4$. An object with different visco elastic parameters at the contact points is considered. The stiffness matrix is $\mathbf{K} = \mathrm{diag}(\mathbf{K}_1, \mathbf{K}_2), \mathbf{K}_1 = \mathrm{diag}(200N/m, 200N/m)$, $\mathbf{K}_2 = 0.5\mathbf{K}_1$, while the damping one is $\mathbf{B} = \mathrm{diag}(\mathbf{B}_1, \mathbf{B}_2)$, $\mathbf{B}_1 = \mathrm{diag}(66Ns/m, 66Ns/m)$, $\mathbf{B}_2 = 0.5\mathbf{B}_1$; the uniformly distributed link (object) mass and the link length (object radius) are $\mathbf{m}_l = 0.3kg$ ($\mathbf{m}_o = 0.25kg$), $l = 0.3m$ ($R = 0.15m$), respectively. The joint position and velocity (PD) feedback gains are set to $\mathbf{R}_q = \mathrm{diag}(10, 10)$ and $\mathbf{R}_{\dot{q}} = \mathrm{diag}(1, 1)$. The contact point is assumed fixed at a distance $0.9l$ from the joints.

The subspace of rigid–body object motion im (Γ_{uc}) has dimension 1 thus $\mathbf{u}_c$ in (4) is a scalar output. The same happens for the reachable internal force output $\mathbf{t}_i$ in (5).

As regards reference inputs, a step of 0.5N is commanded to the input $\mathbf{t}_{i,ref}$ corresponding to the internal force with zero initial value. The input $\mathbf{u}_{c,ref}$ corresponding to the rigid–body motion of the object is a sinusoid having frequency 0.8rad/sec., amplitude 15cm and phase $(-\pi/2 - 0.8t_d)$rad with an offset of 15cm. The input reference $\mathbf{u}_{c,ref}(t)$ (dash–dotted line in fig. 5) is delayed of $t_d = 2$sec with respect to the internal force step.

The robotic system is linearized at three equilibrium points (2) chosen 'close' to the system trajectory corresponding to the commanded object position inputs, namely

$$\left\{\bar{\tau}_1^T, [\bar{\mathbf{q}}^T, \bar{\mathbf{u}}^T, \bar{\dot{\mathbf{q}}}^T, \bar{\dot{\mathbf{u}}}^T]_1, \bar{\mathbf{y}}_1^T\right\} = \{[0,0], [(0,0)(1.5,3,0)(0,0)(0,0,0)], [0,0,0,0,0,0]\}$$
$$\left\{\bar{\tau}_3^T, [\bar{\mathbf{q}}^T, \bar{\mathbf{u}}^T, \bar{\dot{\mathbf{q}}}^T, \bar{\dot{\mathbf{u}}}^T]_3, \bar{\mathbf{y}}_3^T\right\} = \{[0,0], [(-\pi/6, -\pi/6)(3, 2.59, 0)(0,0)(0,0,0)], [0,0,0,0,0,0]\}$$
$$\left\{\bar{\tau}_2^T, [\bar{\mathbf{q}}^T, \bar{\mathbf{u}}^T, \bar{\dot{\mathbf{q}}}^T, \bar{\dot{\mathbf{u}}}^T]_2, \bar{\mathbf{y}}_2^T\right\} = \{[0,0], [(-\pi/3, -\pi/3)(4.1, 1.5, 0)(0,0)(0,0,0)], [0,0,0,0,0,0]\}$$

The linearized models $(\mathbf{A}_{f,p}, \mathbf{B}_{\tau,p}, \mathbf{E}_p)$, $(p = 1,2,3;)$, evaluated at the equilibrium points, are used to build the multiestimator. Regarding the multicontroller $\mathcal{C}_p$, the fixed part $K(s)$ in fig. 1 is a simple proportional controller and is set to diag(1, 10). Observe that $K(s) = K_0$ stabilizes the linearized models at all the 3 equilibrium points. As regards the multiestimator, the performance generator and the switching logic, we have that the stability margin of A_E is set to $\lambda_E = -1500$; the dynamic behaviour of (15) is set to $\lambda = 100$ and finally the minimum amount of time allowed to elapse between two successive controller switchings (dwell–time) is set to $\tau_D = 0.05$.

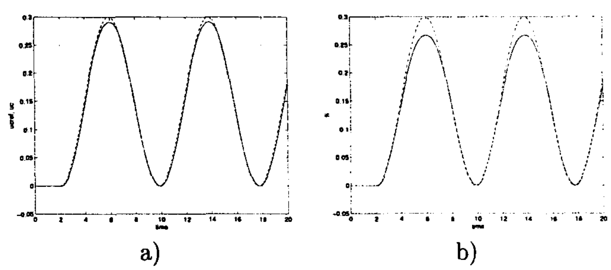


Figure 5: Rigid–body object motions starting from the initial configuration in fig. 4. (a) Logic–based switching control: reference input $\mathbf{u}_{c,ref}$ (dash–dotted line) and system output $\mathbf{u}_c$. (b) Linearized control: reference input $\mathbf{u}_{c,ref}$ (dash–dotted line) and system output $\mathbf{u}_c$.

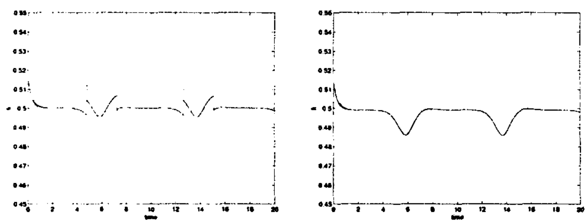
Simulation results of the switching control technique are compared with those obtained without the switching logic. That is, last results are obtained by the same asymptotically decoupling controller but with the switching variable frozen to $\sigma = 1$ (only the linearization at the first equilibrium point has been considered).

Rigid–body object and internal force trajectories are reported in fig. 5 and fig. 6, respectively. Note that the outputs of the switching adaptive system follow their references, in spite of the persistency of the variation of the reference input in fig. 5–a.

In fig. 7 the plot of switching variable σ has been reported together with joint variable $q_1(t)$.

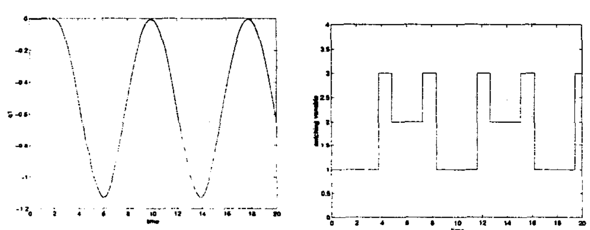
By simple inspection, it clearly appears that the switching logic increases the system performances in position and force tracking.

Observe that the nonlinear system does not pass through the equilibrium points employed to compute the linearized models. This is mainly due to the continuous variations of position inputs and to the fact that the internal force is zero at all the equilibrium points.



a) b)

Figure 6: Reachable internal force behaviour when the reference input is a step of 0.5N. To the initial configuration (fig. 4), It corresponds a zero internal force. (a) Logic–based switching. (b) Linearized control with $\sigma = 1$.



a) b)

Figure 7: In fig. a) it is reported the plot of the first joint variable $q_1(t)$ while in fig. b) it is reported the plot of the switching variable σ. Note that to $q_1 = 0$ ($q_1 = -\pi/6$) $\{q_1 = -\pi./3\}$, it corresponds a linearized model with $\sigma = 1$ ($\sigma = 3$) $\{\sigma = 2\}$.

It is worth stressing that in this section, for the sake of brevity, we have assumed to possess an exact knowledge of the system model on which it is based the decoupling control in fig. 1. Normally, this is not the case and parameter uncertainties should be considered. Observe that taking into account uncertainties is an easy matter in the framework of the multiestimator–based switching control, [6].

6 Conclusions

An application of a logic-based switching controller to general robotic manipulators has been developed and its performance successfully shown via simulations. The multicontroller switches among several controllers designed for linear approximation of the robotics nonlinear dynamics. The aim is to obtain an asymptotic tracking of the object motion and internal force in a co–operative grasp of a single object.

The main contribution and motivation of this work consists in using the logic–based switching control to steer the nonlinear dynamics of general manipulation systems through 'local' controllers with no known global properties. An important issue that must be settled to synthesize the proposed logic–based switching control is the choice of the equilibrium points. At this stage of the work we simply select a set of equilibrium points 'close' to the nominal trajectory. Further investigation is needed to address some criterion of optimality in choosing the linearized models. Currently the research is addressed towards the systematic investigation of the full characterization of the system salient global properties.

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