Force and Dynamic Manipulability for Cooperating Robot Systems

Antonio Bicchi* Domenico Prattichizzo* Claudio Melchiorri**

*Centro "E. Piaggio", Università di Pisa, via Diotisalvi 2, Pisa, Italia
www.piaggio.ccii.unipi.it, bicchi@piaggio.ccii.unipi.it, prattichizzo@dsea.unipi.it

** LAR-DEIS, Università di Bologna, viale Risorgimento 2, Bologna, Italia
www-lar.deis.unibo.it, cmelchiorri@deis.unibo.it

Abstract

A theory of force and dynamic manipulability for general systems of multiple co-operating robot manipulators is developed. Manipulability analysis refers to the study of the performance of the system regarded as a mechanical transformer of inputs (forces and torques at actuated joints) into outputs (forces and torques exchanged with the environment or accelerations of a reference member), in relation with different configurations of the system and different directions in the input and output spaces. To this purpose, the concept of manipulability ellipsoids for single robot arms is generalized so as to encompass multi-limb co-operating systems with general kinematic structure.

1 Introduction

The study of the performance of robotic systems, regarded as mechanical transformers of inputs into outputs, is known as *manipulability* analysis. When velocities of actuated joints are considered as inputs, and velocities of a reference member (the end-effector or the manipulated object) are considered as outputs, one has the so-called kinematic manipulability problem, whose study was pioneered by Yoshikawa [13]. Force manipulability is the dual study of a mechanism regarded as transformer of joint torques to forces exchanged with the environment through the reference member, see [14]. Finally, dynamic manipulability studies the relationship between joint torques as inputs and accelerations of the reference member, [15].

The generalization of kinematic manipulability analysis to co-operating systems of robots has been brought about by [6] and [3]. Those papers assume that each co-operating arm has a number of actuated joints at least equal to the minimum necessary to exchange arbitrary forces with the manipulated object at the contact point. A more general approach for the kinematic manipulability analysis, allowing interaction of co-operating arms with the manipulated object at any point of the kinematic chain (whole-limb manipulation), has been presented in [2].

First results for this type of analysis in the force domain have been illustrated in [1] (considering contact compliance) and in [8, 9] (neglecting it). Some of these results are here used and extended in order to further develop the study both in the force and in the dynamic domain.

In particular, in Section 2 the derivation of the static and dynamic model of a whole-limb manipulator is presented. On this basis, the definition given in [2] of kinematic manipulability is applied to the static (Section 3) and to the dynamic case (Section 4). Examples are discussed in Section 5, while final comments and conclusions are given in Section 6.

2 Background

2.1 Quasi-static model

Let an object be grasped by means of n contacts and let the components of contact forces and moments on the object form a vector $\mathbf{t} \in \mathbb{R}^t$. The type and number of contact force components may vary with the different types of contact considered (hard-finger, soft-finger, complete-constraint, etc.). Consider the task of resisting an external force $\mathbf{f}$ and moment $\mathbf{m}$ applied upon the object. The force and moment balance equation for the object can be written in matrix notation as

$$\mathbf{w} = -\mathbf{G}\mathbf{t}, \tag{1}$$

where $\mathbf{w} = (\mathbf{f}^T\mathbf{m}^T)^T \in \mathbb{R}^d$ is the so-called load "wrench" ($d = 3$ in the plane, and $d = 6$ in the 3D space), and $\mathbf{G} \in \mathbb{R}^{d \times t}$ is usually termed as the "grasp matrix", or "grip transform". We assume that matrix $\mathbf{G}$ is full row rank ($\text{rank}(\mathbf{G}) = d$), so that the existence of a solution to (1) for any $\mathbf{w}$ is guaranteed. In general, (1) has more unknowns (t) than equations (d), so that the solution is not unique.

The relationship between the contact forces on the

Proc. IROS 97 0-7803-4119-8/97/$10©1997 IEEE

fingers and the vector $\tau \in \mathbb{R}^q$ of joint actuator torques is also linear:

$$\tau = \mathbf{J}^T \mathbf{t} \tag{2}$$

where matrix $\mathbf{J} \in \mathbb{R}^{t \times q}$ is the "Jacobian" of the manipulation system. A robot system with $q >$ rank $(\mathbf{J}) \leq t$ (i.e. $\ker(\mathbf{J}) \neq \{\mathbf{0}\}$) is a "redundant" system, while if $t >$ rank $(\mathbf{J}) \leq q$ (i.e. $\ker(\mathbf{J}^T) \neq \{\mathbf{0}\}$), the robot system is "defective" with respect to its contact force space dimension. Exact definitions of the above vectors and matrices can be found in [2, 11, 12].

By juxtaposing quasi-static relationships (1) and (2), one gets

$$\begin{bmatrix} \mathbf{w} \\ \tau \end{bmatrix} = \begin{bmatrix} -\mathbf{G} \\ \mathbf{J}^T \end{bmatrix} \mathbf{t}. \tag{3}$$

When the system is such that matrix $\left[-\mathbf{G}^T \; \mathbf{J}\right]^T$ has a nullspace, i.e. when $\ker(\mathbf{G}) \cap \ker(\mathbf{J}^T) \neq \{\mathbf{0}\}$, the system is "hyperstatic". In such case, it is well known that the contact force distribution problem is underdetermined, and that a model of the system compliance is needed to solve the indeterminacy. This problem can be analyzed as a controllability question as discussed in a quasi-static setting in [1], and in a complete dynamic context in [10, 12]. The interested reader is referred to these papers also for a detailed discussion of force distribution in defective manipulation systems.

To address force distribution problems in hyperstatic mechanisms, mechanical compliance of the system has to be taken into account. Thus, a lumped parameter model of contact compliance is introduced in the quasi-static model by making use of "virtual" springs relating contact forces $\mathbf{t}$ with joint and object displacements.

We describe the system around a reference equilibrium configuration, with contact force $\mathbf{t}_o$, external wrench $\mathbf{w}_o = -\mathbf{G}\mathbf{t}_o$ and joint torque $\tau_o = \mathbf{J}^T\mathbf{t}_o$. As small displacements from the reference configuration are considered, $\mathbf{J}$ and $\mathbf{G}$ are considered constant matrices. We further assume that joints are position-controlled, and that the inverse of the steady-state gain of the i-th joint position servo controller is placed in the i-th diagonal element of a positive definite $q \times q$ stiffness matrix $\mathbf{C}_q$. We denote by $\mathbf{q} = \mathbf{q}_o + \delta\mathbf{q}$ the vector of joint positions, and by $\bar{\mathbf{q}} = \bar{\mathbf{q}}_o + \delta\bar{\mathbf{q}}$ the vector of set points for the joint servos, $\mathbf{q}_o$ and $\bar{\mathbf{q}}_o$ denoting the joint positions and servo set-points at the reference configuration, respectively. At the reference configuration, it holds $\tau_o = \mathbf{C}_q^{-1}(\bar{\mathbf{q}}_o - \mathbf{q}_o)$. We further denote by $\mathbf{u} = \mathbf{u}_o + \delta\mathbf{u}$ the posture of the manipulated object, where $\mathbf{u}_o$ is the object's reference configuration.

Contact forces are obtained as

$$\mathbf{t} = \mathbf{K}(\mathbf{G}^T\delta\mathbf{u} - \mathbf{J}\delta\bar{\mathbf{q}}) + \mathbf{t}_o \tag{4}$$

where the stiffness matrix $\mathbf{K} \in \mathbb{R}^{t \times t}$ incorporates the structural elasticity of the object and of the fingers,

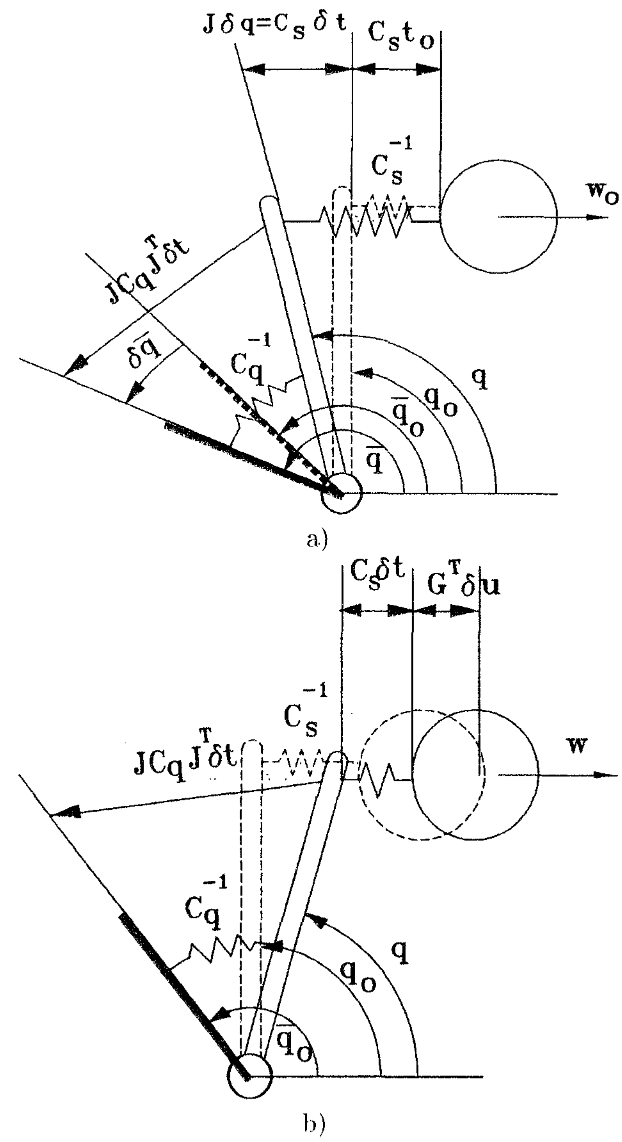

Figure 1: The origin of equation 5. (a) Forces and displacements (linearly superposed) generated by a servo set-point displacement with fixed object position. (b) Forces and displacements generated by an object displacement with fixed servo set-points.

and the stiffness of joint servos [7]. Denoting with $\mathbf{C}_s$ the $t \times t$ structural compliance matrix (due e.g. to the flexibility of links and mechanical transmission, or to soft gripping surfaces), we have that

$$\mathbf{K} = (\mathbf{C}_s + \mathbf{J}\mathbf{C}_q\mathbf{J}^T)^{-1}. \tag{5}$$

As a consequence of its physical nature, $\mathbf{K}$ is assumed symmetric and positive definite. A pictorial representation of the variables involved in (5) is reported in Fig. 1. A detailed and comprehensive study on the evaluation and the realization of desirable stiffness matrices with articulated hands has been presented by

Cutkosky and Kao [4].

The elastic model of contacts (4) enables us to answer the passive force manipulability problem at once. In fact, it can be proven (see e.g. [1]) that contact forces balancing a wrench applied by the environment on the object $\mathbf{w}$ with fixed joint servo set-points are given by

$$\mathbf{t} = \mathbf{G}_K^R \mathbf{w} + \mathbf{E}\mathbf{x} + \mathbf{P}\mathbf{y}, \tag{6}$$

where $\mathbf{G}_K^R = \mathbf{K}\mathbf{G}^T(\mathbf{G}^T\mathbf{K}\mathbf{G}^T)^{-1}$ is the $\mathbf{K}$-weighted pseudoinverse of the grasp matrix; $\mathbf{E}$ is a basis matrix of the subspace of *active, internal* contact forces, i.e. a collection of independent vectors spanning the range of $(\mathbf{I} - \mathbf{G}_K^R\mathbf{G})\mathbf{K}\mathbf{J}$, and $\mathbf{P}$ is a basis matrix of the subspace of *passive, internal* contact forces, i.e. the nullspace of $\left[-\mathbf{G}^T \ \mathbf{J}\right]^T$. Recall that contact forces are said "internal" if they are self-balanced, and do not generate any resultant wrench; "active" if they can be imposed to the system by suitably controlling the joint variables, and "passive" otherwise.

In (6), the coefficient vector $\mathbf{x} \in \mathbb{R}^h$ parameterizes the active (controllable) part of the homogeneous solution to $\mathbf{w} = \mathbf{G}\mathbf{t}$. For any choice of $\mathbf{x}$, a vector of contact forces results that equilibrates the desired load. Vector $\mathbf{x}$ represents the freedom in the choice of internal contact forces, usually exploited to avoid slippage of contacts. In fact, common contact models do not allow arbitrary forces to be exchanged at the contacts, and conic Coulomb inequalities between components of $\mathbf{t}$ are in order.

Such a conic constraints can not be dealt with by methods based on norms such as manipulability ellipsoids. Therefore, in this paper we assume that only bilateral contact constraints are in effect (actually, this assumption is no overwhelmingly restrictive, as we are assuming to deal with small displacements from an equilibrium grasp, where contact force inequalities are verified with some margin).

According to (2), to exert the balancing contact forces, the joint actuators have to apply a torque

$$\tau = -\mathbf{J}^T\mathbf{G}^R\ \mathbf{w} + \mathbf{J}^T\mathbf{E}\mathbf{x}. \tag{7}$$

2.2 Dynamic model

In order to address the dynamic manipulability analysis, a model of the system dynamics is needed.

A robot-object system is a constrained mechanical system, whose dynamics can be obtained by using Euler-Lagrange's equations along with constraint equations. The disjoint dynamics of the hand and of the object are written as

$$\begin{aligned}\left(\frac{d}{dt}\frac{\partial L_h(\mathbf{q},\dot{\mathbf{q}})}{\partial \dot{\mathbf{q}}} - \frac{\partial L_h(\mathbf{q},\dot{\mathbf{q}})}{\partial \mathbf{q}}\right)^T &= \mathbf{M}_h(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{Q}_h(\mathbf{q},\dot{\mathbf{q}}) = \tau;\\ \left(\frac{d}{dt}\frac{\partial L_o(\mathbf{u},\dot{\mathbf{u}})}{\partial \dot{\mathbf{u}}} - \frac{\partial L_o(\mathbf{u},\dot{\mathbf{u}})}{\partial \mathbf{u}}\right)^T &= \mathbf{M}_o(\mathbf{u})\ddot{\mathbf{u}} + \mathbf{Q}_o(\mathbf{u},\dot{\mathbf{u}}) = \mathbf{w},\end{aligned}$$

where $L_h(\cdot,\cdot)$ and $L_o(\cdot,\cdot)$ are the Lagrangians, $\mathbf{M}_h(\cdot)$ and $\mathbf{M}_o(\cdot)$ are symmetric and positive definite inertia matrices and $\mathbf{Q}_h(\cdot,\cdot)$ and $\mathbf{Q}_o(\cdot,\cdot)$ are terms including velocity-dependent and gravity forces of the robotic system and of the object, respectively. Co-operative robots and object dynamics are linked through n contact constraints of the type:

$$\mathbf{C}(\mathbf{q},\mathbf{u})\begin{bmatrix}\dot{\mathbf{q}}\\ \dot{\mathbf{u}}\end{bmatrix} = \begin{bmatrix}\mathbf{J} & -\mathbf{G}^T\end{bmatrix}\begin{bmatrix}\dot{\mathbf{q}}\\ \dot{\mathbf{u}}\end{bmatrix} = \mathbf{0}. \tag{8}$$

By introducing an undetermined t-dimensional vector of Lagrange multipliers $\mathbf{t}$, the virtual work of the connected system can be written as ($L_{ho} = L_h + L_o$)

$$\left[\frac{d}{dt}\frac{\partial L_{ho}}{\partial(\dot{\mathbf{q}},\dot{\mathbf{u}})} - \frac{\partial L_{ho}}{\partial(\mathbf{q},\mathbf{u})} + \mathbf{t}^T\mathbf{C} - \left[\tau^T\ \mathbf{w}^T\right]\right]\begin{bmatrix}\delta\mathbf{q}\\ \delta\mathbf{u}\end{bmatrix} = \mathbf{0},$$

whence, differentiating (8), one gets

$$\mathbf{M}_{dyn}\begin{bmatrix}\ddot{\mathbf{q}}\\ \ddot{\mathbf{u}}\\ \mathbf{t}\end{bmatrix} = \begin{bmatrix}\tau - \mathbf{Q}_h\\ \mathbf{w} - \mathbf{Q}_o\\ \mathbf{Q}_c\end{bmatrix}, \tag{9}$$

where

$$\mathbf{M}_{dyn} = \begin{bmatrix}\mathbf{M}_h & \mathbf{0} & \mathbf{J}^T\\ \mathbf{0} & \mathbf{M}_o & -\mathbf{G}\\ \mathbf{J} & -\mathbf{G}^T & \mathbf{0}\end{bmatrix}, \tag{10}$$

and $\mathbf{Q}_c = \dot{\mathbf{J}}\dot{\mathbf{q}} - \dot{\mathbf{G}}^T\dot{\mathbf{u}}$.

For non-hyperstatic systems, matrix $\mathbf{M}_{dyn}$ is invertible. In such a case, the above rigid-body model is sufficient to completely describe the motion of the system along with the Lagrange multipliers $\mathbf{t}$. Notice that physical interpretation of $\mathbf{t}$ as the force reactions, exactly necessary to enforce the constraints, lead directly to the identity of Lagrange multipliers with contact forces as introduced in the quasi-static model.

For hyperstatic systems, $\mathbf{M}_{dyn}$ is not invertible, and the elastic model (4) has to be adjoined to equation (9) to achieve a complete model of the dynamics.

3 Force manipulability

In the force domain, input efforts are measured by weighted norms of joint torques τ, and output performance is expressed in terms of weighted norms of wrenches $\mathbf{w}$. To arrive at defining force manipulability ellipsoids, we choose weighted 2-norms defined as

$$\begin{aligned}\|\tau\| &= \sqrt{\tau^T\mathbf{W}_\tau\tau};\\ \|\mathbf{w}\| &= \sqrt{\mathbf{w}^T\mathbf{W}_w\mathbf{w}},\end{aligned}$$

with $\mathbf{W}_\tau$ and $\mathbf{W}_w$ constant, positive definite weighting matrices of suitable size and physical dimensions. An efficiency index can then be defined as the square of the ratio

$$R_f^2(\mathbf{t}) = \frac{\|\mathbf{w}\|^2}{\|\tau\|^2} = \frac{\mathbf{t}^T\mathbf{G}^T\mathbf{W}_w\mathbf{G}\mathbf{t}}{\mathbf{t}^T\mathbf{J}\mathbf{W}_\tau\mathbf{J}^T\mathbf{t}}, \tag{11}$$

where (1) and (2) have been used. It must be noted however that maximization of the above quotient cannot be carried over the whole t-dimensional space where the vector of contact forces $\mathbf{t}$ takes its values. In fact, $\mathbf{t}$ is subject to restrictions due to (3).

Another key point in the discussion of force ellipsoids for multiple whole-limb robots concerns how contact forces are generated in the mechanism. In fact, the question behind the manipulability analysis in the force domain is twofold:

Problem 1 : find the external wrench of unit magnitude (in a weighted norm sense) that is balanced with minimum expenditure of joint torque effort (*passive force manipulability* problem);

Problem 2 : find the joint torque vector of unit magnitude (in a weighted norm sense) that generates the maximum performance in terms of wrench applied on the environment (*active force manipulability* problem).

The two problems have different solutions in general, because for mechanisms with defective kinematics, it may not be possible to achieve arbitrarily given contact force vectors $\mathbf{t}$, and hence wrenches $\mathbf{w}$.

3.1 Passive force manipulability

The passive force manipulability problem is solved at once in terms of (7). We choose internal forces that minimize the effort to balance $\mathbf{w}$ as

$$\begin{array}{rl} \text{Min}_\tau & \tau^T \mathbf{W}_\tau \tau \\ \text{Subject to} & \tau = -\mathbf{J}^T \mathbf{G}_K^R \mathbf{w} - \mathbf{J}^T \mathbf{E} \mathbf{x}. \end{array}$$

The minimum is obtained for

$$\mathbf{x} = -(\mathbf{E}^T \mathbf{J} \mathbf{W}_\tau \mathbf{J}^T \mathbf{E})^{-1} \mathbf{E}^T \mathbf{J} \mathbf{W}_\tau \mathbf{J}^T \mathbf{G}_K^R \mathbf{w}$$

and the corresponding optimal solution is

$$\hat{\tau} = -\mathbf{J}^T \hat{\mathbf{G}}^R \mathbf{w},$$

where $\hat{\mathbf{G}}^R = (\mathbf{I} - \mathbf{E}(\mathbf{E}^T \mathbf{J} \mathbf{W}_\tau \mathbf{J}^T \mathbf{E})^{-1} \mathbf{E}^T \mathbf{J} \mathbf{W}_\tau \mathbf{J}^T) \mathbf{G}_K^R$. Consequently, the performance ratio for passive force manipulability can be written as

$$R_{f,p}^2(\mathbf{w}) = \frac{\|\mathbf{w}\|^2}{\|\hat{\tau}\|^2} = \frac{\mathbf{w}^T \mathbf{W}_w \mathbf{w}}{\mathbf{w}^T \hat{\mathbf{G}}^{RT} \mathbf{J} \mathbf{W}_\tau \mathbf{J}^T \hat{\mathbf{G}}^R \mathbf{w}}. \qquad (12)$$

As it is well known, the maximum (minimum) value of the *Rayleigh quotient* in (12) corresponds to the maximum (minimum) eigenvalue of the *pencil* $\mathbf{W}_w - \lambda \hat{\mathbf{G}}^{RT} \mathbf{J} \mathbf{W}_\tau \mathbf{J}^T \hat{\mathbf{G}}^R$. Accordingly, the directions (in the external wrench space) in which a maximum (minimum) efficiency is obtained are given by the generalized eigenvectors corresponding to the maximum (minimum) eigenvalues of the pencil. Details on the efficient computation of generalized eigenvalues are reported e.g. in [5]. If σ_{max} is the maximum eigenvalue and $\mathbf{w}_{max}$ the corresponding eigenvector, the corresponding direction in the joint torque space is given by $-\mathbf{J}^T \hat{\mathbf{G}}^R \mathbf{w}_{max}$.

3.2 Active force manipulability

The active force manipulability problem is only meaningful if the motion of the reference member of the mechanism, i.e. the object, is inhibited by the environment in all directions. In this hypothesis, manipulation systems are always hyperstatic, and an elastic model of the object–environment interaction is mandatory to proceed in the analysis. Let this model be

$$\mathbf{w} = -\mathbf{K}_e \delta \mathbf{u}, \qquad (13)$$

where $\mathbf{K}_e$ is a symmetric, positive definite $d \times d$ environment stiffness matrix. The choice of $\mathbf{K}_e$ will be discussed later on in this section.

Consider the object balance equation (1) and substitute the elastic interaction models (4) and (13),

$$\mathbf{G}\mathbf{K}(\mathbf{G}^T \delta \mathbf{u} - \mathbf{J} \delta \bar{\mathbf{q}}) = -\mathbf{K}_e \delta \mathbf{u},$$

whence the object displacement $\delta \mathbf{u}$ reached at equilibrium after imposing a joint servo set-point change $\delta \bar{\mathbf{q}}$ is obtained as

$$\delta \mathbf{u} = (\mathbf{G}\mathbf{K}\mathbf{G}^T + \mathbf{K}_e)^{-1} \mathbf{G}\mathbf{K}\mathbf{J} \delta \bar{\mathbf{q}} \stackrel{def}{=} \mathbf{T}\mathbf{J}\delta \bar{\mathbf{q}}.$$

Substituting in (13) and (4), and using (2), we have

$$\begin{array}{rcl} \mathbf{w} & = & -\mathbf{K}_e \mathbf{T} \mathbf{J} \delta \bar{\mathbf{q}} \\ \tau & = & -\mathbf{J}^T \mathbf{K} (\mathbf{G}^T \mathbf{T} - \mathbf{I}) \mathbf{J} \delta \bar{\mathbf{q}}. \end{array}$$

Finally, the Rayleigh quotient (11) for active force manipulability is evaluated as

$$\mathbf{R}_{f,a}^2(\delta \bar{\mathbf{q}}) = \frac{\delta \bar{\mathbf{q}}^T \mathbf{N} \delta \bar{\mathbf{q}}}{\delta \bar{\mathbf{q}}^T \mathbf{D} \delta \bar{\mathbf{q}}} \qquad (14)$$

where

$$\begin{array}{rl} \mathbf{N} = & \mathbf{J}^T \mathbf{T}^T \mathbf{K}_e \mathbf{W}_w \mathbf{K}_e \mathbf{T} \mathbf{J} \\ \mathbf{D} = & \mathbf{J}^T (\mathbf{T}^T \mathbf{G} - \mathbf{I}) \mathbf{K} \mathbf{J} \mathbf{W}_\tau \mathbf{J}^T \mathbf{K} (\mathbf{G}^T \mathbf{T} - \mathbf{I}) \mathbf{J}. \end{array}$$

Again, the maximum (minimum) value of the *Rayleigh quotient* in (14) corresponds to the maximum (minimum) eigenvalue of the pencil $\mathbf{N} - \lambda \mathbf{D}$. Accordingly, the directions (in the joint servo set-point space) in which a maximum (minimum) efficiency is obtained are given by the generalized eigenvectors corresponding to the maximum (minimum) eigenvalues of the pencil. If σ_{max} is the maximum eigenvalue and $\bar{\mathbf{q}}_{max}$ its eigenvector, the corresponding direction in the joint torque and external wrench space are given by $\tau = -\mathbf{J}^T \mathbf{K} (\mathbf{G}^T \mathbf{T} - \mathbf{I}) \mathbf{J} \delta \bar{\mathbf{q}}_{max}$ and $\mathbf{w} = -\mathbf{K}_e \mathbf{T} \mathbf{J} \delta \bar{\mathbf{q}}_{max}$.

From (14) it is also clear how the external stiffness matrix $\mathbf{K}_e$ only enters in the product $\mathbf{K}_e \mathbf{W}_w \mathbf{K}_e$, thus effectively playing the role of a wrench weight matrix.

4 Dynamic manipulability ellipsoids

According to Yoshikawa's definition of dynamic manipulability, we consider the ability of a co-operating manipulation system to accelerate the reference member of the chain (the object in our case) from an equilibrium configuration with $\dot{\mathbf{q}} = \dot{\mathbf{u}} = \mathbf{0}$.

In this case, dynamics in (9) simplifies to

$$\mathbf{M}_{dyn} \begin{bmatrix} \ddot{\mathbf{q}} \\ \ddot{\mathbf{u}} \\ \mathbf{t} \end{bmatrix} = \begin{bmatrix} \tau - \mathbf{g}_h \\ \mathbf{w} - \mathbf{g}_o \\ \mathbf{0} \end{bmatrix},$$

where $\mathbf{g}_h, \mathbf{g}_o$ are gravity terms acting on the limbs and on the object, respectively.

We assume in this section that the system is not hyperstatic, so that a rigid-body model will only be needed for the analysis.

Under this assumption, the inverse of the dynamic matrix $\mathbf{M}_{dyn}$ can be explicitly computed. In fact, defining matrices $\mathbf{A}$ and $\mathbf{B}$ as

$$\mathbf{A} = \begin{bmatrix} \mathbf{M}_h & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_o \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} \mathbf{J}^T \\ -\mathbf{G} \end{bmatrix},$$

and noting that $\mathbf{A}$ and $\mathbf{B}^T\mathbf{B}$ are symmetric and positive definite matrices, one gets

$$\mathbf{M}_{dyn}^{-1} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{0} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{L}_{11} & \mathbf{L}_{12} \\ \mathbf{L}_{21} & \mathbf{L}_{22} \end{bmatrix};$$

with

$$\begin{aligned} \mathbf{L}_{11} &= \mathbf{A}^{-1}\left(\mathbf{I} - \mathbf{B}(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{A}^{-1}\right); \\ \mathbf{L}_{12} &= \mathbf{B}\mathbf{A}^{-1}(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1}; \\ \mathbf{L}_{21} &= (\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{A}^{-1}; \\ \mathbf{L}_{22} &= -(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1}. \end{aligned}$$

By partitioning the upper-left block of this matrix as

$$\mathbf{A}^{-1}\left(\mathbf{I} - \mathbf{B}(\mathbf{B}^T\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{A}^{-1}\right) = \begin{bmatrix} \mathbf{X}_{q\tau} & \mathbf{X}_{qw} \\ \mathbf{X}_{u\tau} & \mathbf{X}_{uw} \end{bmatrix}$$

such that $\mathbf{X}_{q\tau} \in \mathbb{R}^{(q+d)\times(q+d)}$ and so forth, one obtains the solution of the dynamics in terms of object accelerations as

$$\ddot{\mathbf{u}} = \mathbf{X}_{u\tau}(\tau - \mathbf{g}_h) + \mathbf{X}_{uw}(\mathbf{w} - \mathbf{g}_o).$$

Thus, the dynamic manipulability Rayleigh ratio (for zero external wrench) is written as

$$\mathbf{R}_d^2(\tau) = \frac{\|\ddot{\mathbf{u}}\|}{\|\tau\|} = \frac{(\tau - \mathbf{g}_h)^T\mathbf{X}_{u\tau}^T\mathbf{W}_{\ddot{u}}\mathbf{X}_{u\tau}(\tau - \mathbf{g}_h)}{\tau^T\mathbf{W}_\tau\tau}.$$

Discussion on generalized eigenvalue problems reported above apply here as well, with the observation that the effect of the gravity term $\mathbf{g}_h$ corresponds to displace the center of the ellipsoid by vector $\mathbf{g}_h$. Non-zero wrenches applied on the object have similar effects on dynamic manipulability.

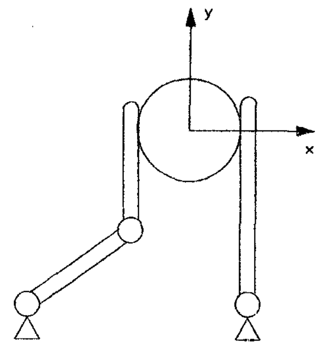

Figure 2: Example of cooperating manipulation.

Finally, in some applications it may be important to choose a configuration on the basis of how well the object motions are actuated not by joints, but rather by external wrenches. Thus the suitable ratio to be studied becomes

$$R_d^2(\mathbf{w}) = \frac{\|\ddot{\mathbf{u}}\|}{\|\mathbf{w}\|} = \frac{(\mathbf{w} - \mathbf{g}_o)^T\mathbf{X}_{uw}^T\mathbf{W}_{\ddot{u}}\mathbf{X}_{uw}(\mathbf{w} - \mathbf{g}_o)}{\mathbf{w}^T\mathbf{W}_w\mathbf{w}}.$$

5 Examples

Consider the simple two-arm cooperating system schematically represented in Fig. 2, consisting of a two-d.o.f. and a one-d.o.f. mechanism. An object is held between the distal links of the arms, by means of two 'soft finger' contacts at $(-0.25, 0)$ and $(0.25, 0)$. The passive force manipulability analysis is applied to this case with identity weight matrices for simplicity. Also note that, because the system is non-hyperstatic, the value of the stiffness matrix $\mathbf{K}$ does not affect the results of this analysis. The resulting ellipsoid in the wrench space (which is three-dimensional for this 2D mechanism) is almost degenerate to a segment (eigenvalues are $\begin{bmatrix} -2.2e-017 & 2.7e-016 & 1.1 \end{bmatrix}$), whose direction is

$$v_{max} = \begin{bmatrix} 4 \\ -1 \\ 4 \end{bmatrix}.$$

Wrenches in this direction cause the largest effort on the actuators (i.e., minimize the Rayleigh ratio $R_{f.p}(w)$) while wrenches in the orthogonal directions (orthogonality is defined modulo $\mathbf{W}_w$) are passively resisted by the mechanism.

It is interesting to notice that, if the internal force were not optimized, the ellipsoid would have had eigenvalues $\begin{bmatrix} -5e-017 & 0.2 & 1.7 \end{bmatrix}$, i.e., the mechanism would use more actuator torques to resist the same load.

To apply the active force manipulability analysis to the same mechanism, specification of internal ($\mathbf{K}$) and environment ($\mathbf{K}_e$) stiffness matrices is necessary. For simplicity, we let them be 4 by 4 and 3 by 3 identity matrices, respectively (changing these values does affect the ensuing results). From the formulas of section 3.2, the active manipulability ellipsoid in the joint space (which is three-dimensional in our example) has eigenvalues $\begin{bmatrix} 0.75 & 71 & 85 \end{bmatrix}$, corresponding to directions

$$v_{min} = \begin{bmatrix} -1 \\ 16 \\ 8.9 \end{bmatrix}, v_{saddle} = \begin{bmatrix} 1.7 \\ -1.8 \\ -1 \end{bmatrix}, v_{max} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}.$$

These results clearly confirm the intuitive inadequacy of the mechanism to apply forces in the y-direction.

6 Conclusions

A theory of force and dynamic manipulability for general systems of multiple co-operating robot manipulators is proposed. It extends the concept of manipulability ellipsoids for single robot arms to encompass multi-limb co-operating systems with general kinematic structure. Future work will concern extensions to hyperstatic dynamic manipulability, and generalizations to underactuated joints.

References

[1] A. Bicchi, "On the Problem of Decomposing Grasp and Manipulation Forces in Multiple Whole-Limb Manipulation", Int. J. on Robotics and Autonomous Systems, Vol. 13, pp. 127-147, 1993.

[2] A. Bicchi, C. Melchiorri, and D. Balluchi "On the Mobility and Manipulability of General Multiple Limb Robots", IEEE Trans. on Robotics and Automation, Vol. 11, No. 2, pp. 215–228, 1995.

[3] P. Chiacchio, S. Chiaverini, L. Sciavicco, B. Siciliano, "Global Task Space Manipulability Ellipsoids for Multiple-Arm Systems", *IEEE Trans. on Robotics and Automation*, Vol. 7, No. 5, Oct. 1991.

[4] Cutkosky, M.R., and Kao, I.: "Computing and Controlling the Compliance of a Robotic Hand", IEEE Trans. on Robotics and Automation, Vol. 5, No. 2, 1989.

[5] G.H. Golub and C.F. VanLoan, *"Matrix Computations"*, Johns Hopkins University Press, Baltimore, MD, 1989.

[6] S. Lee, "Dual Redundant Arm Configuration Optimization with Task-Oriented Dual Arm Manipulability", *IEEE Trans. on Robotics and Automation*, Vol. 5, No. 1, 1989.

[7] M.T. Mason, J.K. Salisbury, "Robot Hands and the Mechanics of Manipulation," MIT Press, Cambridge, MA, 1985.

[8] C. Melchiorri, "Multiple Whole-Limb Manipulation: an Analysis in the Force Domain", *Int. J. on Robotics and Autonomous Systems*. Accepted March 1997. To appear.

[9] T. Omata, K. Nagata, "Rigid Body Analysis of Power Grasping", Proc. 1995 IEEE Int. Conf. on Robotics and Automation, Japan, May, 1995.

[10] D. Prattichizzo, A. Bicchi, "Specifying consistent control goals for kinematically defective manipulation systems," in *IEEE Int. Conf. Robotics Automat.*, Minneapolis, 1996.

[11] D. Prattichizzo, A. Bicchi, "Dynamic analysis of mobility and graspability of general manipulation systems," in *IEEE Transactions on Robotics and Automation*. To appear. Accepted 1997. Available from WWW.

[12] D. Prattichizzo and A. Bicchi, "Consistent task specification for manipulation systems with general kinematics," in *ASME Journ. Dyn. Sys., Meas., and Control* To appear. Accepted 1997. Available from WWW.

[13] T. Yoshikawa, "Analysis and Control of Robot Manipulators with Redundancy", in *Prep. 1st. Int. Symp. Robotics Research*, Bretton Woods, N.H., 1983.

[14] T. Yoshikawa, "Manipulability of Robotics Mechanisms", *Int. Jour. of Robotics Research*, Vol. 4, No. 2, 1985.

[15] T. Yoshikawa, "Dynamic Manipulability of Robot Manipulators", *Jour. of Robotic Systems*, Vol. 2, No. 1, 113-124, 1985.