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Controllability of Whole–Arm Manipulation

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Abstract

Whole–Arm Manipulation systems (WAMs), i.e. robotic devices that use not only their extremities but rather any of their links to manipulate objects, are focusing much attention recently because of their robustness and wide range of applicability. The analysis and control of such systems pose problems not common to traditional robotics. In this paper, we consider a peculiar loss of a controllability property that deeply affects operation of WAMs.

1 Introduction

Whole–Arm Manipulation systems (WAMs — [Salisbury, 1987]) are robotic devices that manipulate objects by exploiting also their inner links and parts. One of the purposes of WAMs is to overcome the limitations that impose very low payload–to–weight ratios to conventional arms.

The analysis and control of WAM systems pose new problems with respect to more traditional robotic devices. The most important peculiarity is that inner links generally have fewer degrees–of–freedom than necessary to achieve arbitrary configurations in their operational space, i.e., are kinematically defective. This fact entails several differences in the treatment of the kinematics, statics, dynamics, and control. In this paper, we focus on the problem of dynamic manipulation control, i.e., how can the system control contact forces between links and the object, so as to have the object track a desired trajectory while balancing external forces, and obeying to nonlinear inequalities representing physical constraints on the task. When dealing with WAMs, achieving such goal may not be possible for arbitrarily assigned trajectories in position and force. It is therefore crucial to understand what feasibility conditions are imposed on desired manipulation tasks by the given manipulator structure. This problem has two main aspects, a force–distribution and a kinematic one. Previous work on these subproblems has been done by Bicchi [1994] and by Bicchi, Melchiorri, and Balluchi [1993], respectively, in a quasi–static setting. In the present paper we approach the problem dynamically, find previous results as particular cases, and present a new result concerning the correct specification of control objectives that provides a basis for synthesizing hybrid (position/force) controllers of WAMs.

2 Dynamic Model

A cooperative WAM system (e.g., a hand) is a constrained mechanical system, whose dynamical description can be derived using Lagrange's equations together with constraint equations. Let $\mathbf{q} \in \mathbb{R}^q$ denote the vector of joint positions, and let $\mathbf{u} \in \mathbb{R}^d$ be the vector locally describing the position and orientation (using e.g. Euler angles) of a frame attached to the object. The dynamics of the hand and of the object are driven by joint torques τ and external forces/torques $\mathbf{w}$, respectively, and are linked through contact constraint equations, which can be written in matrix form as $\mathbf{G}^T\dot{\mathbf{u}} - \mathbf{J}\dot{\mathbf{q}} = \mathbf{0}$, where $\mathbf{G}$ is usually referred to as "grasp matrix", or "grip transform", while $\mathbf{J}$ is the aggregated hand Jacobian. In non–defective manipulators, introduction of these constraints in the virtual work equation through indeterminate Lagrange multipliers yields the desired model. Due to the inherently hyperstatic nature of WAMs, however, the rigid–body model is inadequate, and consideration of the visco–elasticity of materials involved is mandatory. We adopt a simple phenomenological model of contact compliance and damping, consisting of a set of virtual springs and dampers interposed at each contact point. For simplicity's sake, we assume also that visco–elasticity is linear and isotropic, that its lumped parameters are known (perhaps via identification), and that contact points do not change by rolling.

In this setting, Lagrange multipliers $\mathbf{t}$ can be interpreted as constraint forces deriving from the virtual springs and dampers with endpoints attached at the contact points ${}^h\mathbf{c}_i$ and ${}^o\mathbf{c}_i$, as $\mathbf{t} = \mathbf{K}({}^h\mathbf{c}_i - {}^o\mathbf{c}_i) + \mathbf{B}(\mathbf{J}\dot{\mathbf{q}} - \mathbf{G}^T\dot{\mathbf{u}})$. Accordingly, the hand and object dynamic equations can be written as

$$\ddot{\mathbf{q}} = \mathbf{M}_h^{-1}\left(-\mathbf{Q}_h - \mathbf{J}^T\mathbf{t} + \tau\right); \qquad (1)$$

$$\ddot{\mathbf{u}} = \mathbf{M}_o^{-1}\left(-\mathbf{Q}_o + \mathbf{G}\mathbf{t} + \mathbf{w}\right), \qquad (2)$$

where the $\mathbf{M}_i(\cdot)$ are inertia matrices and the $\mathbf{Q}(\cdot,\cdot)$ terms include velocity-dependent and gravity forces. For the analysis of most of the structural properties of WAM systems, the model (1)–(2) is still intractable. Henceforth, then, we will deal with the linearized dynamic model

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau\tau' + \mathbf{B}_w\mathbf{w}', \qquad (3)$$

where the state vector and inputs are defined as the departures from a reference equilibrium configuration $\mathbf{q}_o$, $\mathbf{u}_o$, $\dot{\mathbf{q}} = \dot{\mathbf{u}} = \mathbf{0}$, $\tau' = \tau - \mathbf{J}^T\mathbf{t}_o$, $\mathbf{w}' = \mathbf{w} + \mathbf{G}\mathbf{t}_o$, and

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{L}_k & \mathbf{L}_b \end{bmatrix}; \mathbf{B}_\tau = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_h^{-1} \\ \mathbf{0} \end{bmatrix}; \ \mathbf{B}_w = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_o^{-1} \end{bmatrix}.$$

Neglecting gravity and assuming that $\mathbf{t}_o$ is small, we have $\mathbf{L}_k = -\mathbf{M}^{-1}\mathbf{P}_k$ and $\mathbf{L}_b = -\mathbf{M}^{-1}\mathbf{P}_b$, where $\mathbf{M} = \mathrm{diag}(\mathbf{M}_h, \mathbf{M}_o)$, $\mathbf{P}_k = \mathbf{S}^T\mathbf{K}\mathbf{S}$, $\mathbf{P}_b = \mathbf{S}^T\mathbf{B}\mathbf{S}$, and $\mathbf{S} = [\mathbf{J} \ -\mathbf{G}^T]$.

3 Controllability of WAM Systems

Defective kinematics of WAM systems involve in general the loss at some extent of structural properties such as pointwise controllability or observability ([Bicchi and Prattichizzo, 1994]). However, to the purposes of the force/position tracking problem set out in the introduction, the property of primary concern here is the functional controllability (f.c.) ([Rosenbrock, 1970]) of outputs defined by $\mathbf{C_t} = [\mathbf{KJ} \ \ -\mathbf{KG}^T \ \mathbf{BJ} \ \ -\mathbf{BG}^T]$, i.e., contact forces. Note that complete f.c. of contact forces (along with the condition that $\mathbf{G}$ is full row rank) guarantees f.c. of object positions. However, kinematically defective systems do not enjoy such property, as can be easily seen by considering the frequency domain, input–output description

$$\mathbf{t}(s) = \mathbf{Z}_{t,\tau}(s)\tau(s) + \mathbf{Z}_{t,w}\mathbf{w}(s); \qquad (4)$$

where

$$\begin{aligned}
\mathbf{Z}_{t,\tau} &= \mathbf{C_t}\,(s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B}_\tau \\
&= (\mathbf{K} + s\mathbf{B})(\mathbf{J} + \mathbf{G}^T\mathcal{D}^{-1}\mathcal{B}^T)(\mathcal{A} - \mathcal{B}\mathcal{D}^{-1}\mathcal{B}^T)^{-1}, \\
\mathcal{A} &= s^2\mathbf{M}_h + s(\mathbf{J}^T\mathbf{BJ} + \mathbf{R}_{\dot q}) + \mathbf{J}^T\mathbf{KJ} + \mathbf{R}_q; \\
\mathcal{B} &= -s\mathbf{J}^T\mathbf{BG}^T - \mathbf{J}^T\mathbf{KG}^T; \\
\mathcal{D} &= s^2\mathbf{M}_o + s\mathbf{GBG}^T + \mathbf{GKG}^T.
\end{aligned}$$

(here $\mathbf{R}_q$, $\mathbf{R}_{\dot q}$ indicate P.D. gains of joint position servos). It is evident from the dimensions of the transfer function matrix $\mathbf{Z}_{t,\tau}$ that contact forces can not be controlled along an arbitrary desired trajectory as long as there are more force components than joints ($\mathbf{J}$ is not f.r.r.). Contact forces on the inner links of WAMs are not f.c.. Given a manipulation systems, it is important to understand what trajectories in $\mathbf{u}$ and in $\mathbf{t}$ can be actually tracked. Most thorough techniques to this end are developed by geometric state-space methods, see e.g. [Basile and Marro, 1971]. Unfortunately, however, the iterative nature of the V^* algorithm does not allow a clear interpretation of results in terms of kinematic and structural parameters of the system. Our approach is therefore to define a new set of outputs that is functionally controllable, exhausts the system capabilities, and is relevant to the task to be performed.

To do so, it is instrumental to consider first asymptotic reproducibility of outputs [Brockett and Mesarovich, 1965]. Assuming that $\mathbf{K}$ and $\mathbf{B}$ are p.d., and that joint position and rates have been fed back such that all modes of the system are asymptotically stable, the steady-state gain matrix is

$$\begin{aligned}
\mathbf{Z}_{t,\tau}(0) &= -\mathbf{C_t}\hat{\mathbf{A}}^{-1}\mathbf{B}_\tau \qquad (5)\\
&= -(\mathbf{I} - \mathbf{G}^+_{\bar K}\mathbf{G})\mathbf{KJ},
\end{aligned}$$

where $\mathbf{G}^+_{\bar K}$ is the $\bar{\mathbf{K}}$-weighted pseudoinverse of $\mathbf{G}$, and $\bar{\mathbf{K}}^{-1} = \mathbf{K}^{-1} + \mathbf{J}\mathbf{R}_q^{-1}\mathbf{J}^T$ is the equivalent stiffness matrix including the effect of proportional control on joint positions (cf. Cutkosky and Kao [1989]). Elements of **range** $(\mathbf{Z}_{t,\tau})$ are called "controllable internal" forces, since they correspond to forces that are in the nullspace of $\mathbf{G}$ and that can be actively used by the controller to avoid constraint violation (e.g., to avoid slippage). This subspace corresponds to that found in [Bicchi, 1994] by quasi-static methods.

Let $\mathbf{E}$ be a basis matrix for the subspace of controllable internal forces. Moreover, let Γ_{qc} and Γ_{uc} be basis matrices for the rigid-body coordinated motions of the system, and Γ_r a basis matrix for redundant motions (see [Bicchi, Melchiorri, and Balluchi, 1993]). Then we have:

Theorem 1 ***A robotic manipulation system with linearized dynamics described by the triple*** $(\mathbf{A}, \mathbf{B}_\tau, \mathbf{C})$***, where*** $\mathbf{A}$ ***and*** $\mathbf{B}_\tau$ ***are as in (3), and output matrix***

$$\mathbf{C} = \begin{bmatrix} \Gamma^+_{uc}\mathbf{C_u} \\ \mathbf{E}^+\mathbf{C_t} \\ \Gamma^+_r\mathbf{C_q} \end{bmatrix}, \qquad (6)$$

is square and functionally controllable.

Proof. See [Bicchi and Prattichizzo, 1994].

4 Discussion

A task-oriented priority in the specification of outputs is reflected in the top-down order of the output matrix $\mathbf{C}$ in (6). In fact, the first group of outputs are coordinates for the subspace of rigid-body displacements of the manipulated object; similarly the second group of outputs for the subspace $\mathcal{F}_{hr}$ of controllable internal contact forces (in the basis $\mathbf{E}$), and the third group for the subspace of redundant degrees-of-freedom (in the basis Γ_r). As a result of theorem 1, the whole of these three subspaces is functionally controllable, and so is their direct sum. While other feasible choices of outputs for traking are possible, the one proposed here is closest to the specifications of a manipulation task.

5 References

Basile, G., and Marro, G.: "On the Perfect Output Controllability of Linear Dynamic Systems", Ricerche di Automatica, vol. 2, no. 1, pp. 1-10, 1971.

A. Bicchi: "On the Problem of Decomposing Grasp and Manipulation Forces in Multiple Whole-Limb Manipulation", *Int. J. of Robotics and Autonomous Systems.* 13 (1994) 127-147. Elsevier Science, 1994.

Bicchi, A., Melchiorri, C., and Balluchi, D.:"On the Mobility and Manipulability of General Multiple Limb Robots", IEEE Trans. Robotics and Automation. In press. 1993.

Bicchi, A., and Prattichizzo, D.: "On Some Structural Properties of Robotic Manipulation Systems", submitted. Available as C.P. Memo 21093, Centro "E.Piaggio", Univ. di Pisa. 1994.

Brockett, R.W., Mesarovich, M.:"The Reproducibility of Multivariable Systems", Jour. Math. Anal. Appl., vol. 11, pp. 548,563, 1965.

Cutkosky, M.R., and Kao, I.: "Computing and Controlling the Compliance of a Robotic Hand", IEEE Trans. on Robotics and Automation, vol.5, no.2, 1989.

Rosenbrock, H.H: " State-Space and Multivariable Theory", Thomas Nelson and Sons, Ltd. 1970.

Salisbury, J. K.: "Whole-Arm Manipulation", Proc. of the 4-th International Symposium of Robotics Research, Santa Cruz, CA. MIT Press, 1987.