Chapter 1

On some structural properties of general manipulation systems

This chapter analyses the geometric and structural characteristics involved in the control of rather general manipulation systems, consisting of multiple cooperating linkages, interacting with a reference member of the mechanism (the "object") by means of contacts on any available part of their links. Object grasp and manipulation by the human hand is taken as a paradigmatic example for this class of manipulators, while classical mechanisms (including closed kinematic chains) can be shown to fit easily in this framework.

We present an analytical formulation of the kinematics and dynamics of such systems. Moreover, we report on some recent results on the analysis and control of these mechanisms, based on a geometric analysis of a local approximation of system dynamics. Notwithstanding the local nature of the latter study, it provides a very insightful view of the problem, along with control techniques that achieve interesting performance.

1.1 Introduction

In the past three decades, research on the geometric approach to dynamic systems theory and control has achieved important results, which made that approach a powerful and thorough tool in the analysis and synthesis of linear systems ([1], [17]). Among the successes of the geometric approach, it must be counted the contribution to the development of a nonlinear systems theory, stemming from and generalizing on deeply geometric ideas ([6]).

On the other hand, in the same years, mechanical systems used in industry and developed in research labs also evolved quite quickly. Robotics is one notable case of such evolution. In response to the stepping–up of requirements on the control of mechanical systems engendered by the tightening of performance specifications, the increase in number of degrees–of–freedom, and the introduction of interacting robotic limbs (as e.g. in pairs of cooperating

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arms, multifingered hands, and legged vehicles), rather sophisticated analysis and control techniques have been developed by the robotics community.

General systems of interacting multiple robot limbs could be used to model arbitrary mechanisms. Typical robotics concepts and tools, such as e.g. manipulability analysis, can then be applied to such system (see [4]). A unified control theory of mechanical systems is conceivable, drawing upon recent results in robotics to extend them to rather general classes of mechanisms. To do this, however, it is necessary that some assumptions on the description of cooperating robot limbs limiting their generality are lifted, and that the corresponding theory is fully understood.

Among the generalizations of robotic models that have to be considered to achieve that goal, are the following:

1. each interacting limb can interact with the object with any of its links;
2. the interaction with the object can be specified by several different models, ranging from rigid attachment to rolling and/or sliding contacts between the bodies, etc.;
3. some of the limb joints may not be actuated.

These generalizations entail non–trivial modifications in the theoretical approach. Tools from geometric control theory are particularly useful in understanding these more general systems. In this chapter, we report on some recent advances towards the goal of a general, unified treatment of manipulation systems.

1.2 Kinematics

The model of the mechanisms we consider is comprised of an arbitrary number of actuated linkages (i.e. simple chains of links, connected through rotoidal or prismatic joints) and of an object which is in contact, at one or more points, with some of the links. We define the vector $\mathbf{q}$ as a vector of generalized coordinates, completely describing the configuration of the limbs; and the vector $\mathbf{u}$ as a generalized coordinate vector for the object.

Contacts represent a particular kind of kinematic constraint on the allowable configurations of the system. Contact constraints are typically unilateral, possibly non-holonomic constraints on the generalized coordinates system, written in general in the form

$$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}},\mathbf{u},\dot{\mathbf{u}}) \geq 0. \tag{1.1}$$

The inequality relationship reflects the fact that contact can be lost if the contacting bodies are brought away from each other. This involves an abrupt change of the structure of the model under consideration. To avoid analytical difficulties, it is usually assumed that the manipulation is studied during time intervals when constraints hold with the equal sign. The constraint relationship (1.1) is not in general integrable, i.e., it cannot be expressed in terms of $\mathbf{q}$ and $\mathbf{u}$ only: integrable constraints are called holonomic. Holonomic constraints between generalized coordinates reduce the number of independent coordinates necessary to describe the system configuration (degrees of freedom), and can conceivably be removed from the description of the system by proper coordinate substitution. Nonholonomic constraints, on the contrary, do not reduce the number of degrees-of-freedom of the system, but rather reduce the number of independent coordinate velocities. A typical example of nonholonomic constrained motion is the rolling of two bodies on top of each other. Nonholonomy introduces many peculiar difficulties in the analysis and control of mechanical manipulation systems, some of which have been addressed in [11, 2]. No results concerning nonholonomic systems will be discussed in this chapter.

Several types of contact models can be used to describe the interaction between the links and the object. When a rigid–body model of the mechanism is considered, the constraints consist in imposing that some components of the relative generalized velocity between two reference frames associated with the contact point on each surface, are zero:

$$\mathbf{H}_i\ ({}^o\dot{\mathbf{c}}_i -{}^f\ \dot{\mathbf{c}}_i) = 0 \tag{1.2}$$

where $\mathbf{H}_i$ is a constant selection matrix. Being the two frames fixed on the object and the phalanx, respectively, their velocities can be expressed as a linear function of the velocities of the object and of the joints as

$$\begin{aligned} {}^o\dot{\mathbf{c}}_i &= \tilde{\mathbf{G}}_i^T(\mathbf{u})\ \dot{\mathbf{u}}; &(1.3)\\ {}^f\dot{\mathbf{c}}_i &= \tilde{\mathbf{J}}_i(\mathbf{q}_i)\ \dot{\mathbf{q}}. &(1.4)\end{aligned}$$

Similar relationships hold for each contact point, and a single equation can be built to represent all constraints by properly juxtaposing vectors and block matrices to obtain

$$\mathbf{H}\tilde{\mathbf{G}}^T\dot{\mathbf{u}} - \mathbf{H}\tilde{\mathbf{J}}\dot{\mathbf{q}} = 0. \tag{1.5}$$

Let $s = 2$, $d = 3$ for 2D mechanisms and $s = 3$, $d = 6$ for 3D ones. Denoting by $\mathbf{p}$ the object center of mass, it holds

$$\begin{aligned} \tilde{\mathbf{G}} &= \left[\begin{array}{ccc|ccc} \mathbf{I}_s & \cdots & \mathbf{I}_s & & \mathbf{0}_{s\times n(d-s)} & \\ \mathbf{S}({}^o\mathbf{c_1}-\mathbf{p}) & \cdots & \mathbf{S}({}^o\mathbf{c_n}-\mathbf{p}) & \mathbf{I}_{d-s} & \cdots & \mathbf{I}_{d-s} \end{array}\right]; \quad \tilde{\mathbf{G}} \in \mathbb{R}^{d\times nd}\\ \tilde{\mathbf{J}}^T &= \left[\begin{array}{ccc|ccc} \mathbf{D}_{1,1} & \cdots & \mathbf{D}_{n,1} & \mathbf{L}_{1,1} & \cdots & \mathbf{L}_{n,1}\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \mathbf{D}_{1,r} & \cdots & \mathbf{D}_{n,r} & \mathbf{L}_{1,r} & \cdots & \mathbf{L}_{n,r} \end{array}\right]; \quad \tilde{\mathbf{J}} \in \mathbb{R}^{nd\times q}; \end{aligned}$$

where

$$\mathbf{S}(\mathbf{c_i}) = \begin{bmatrix} 0 & -c_{i,y} & c_{i,z}\\ c_{i,y} & 0 & -c_{i,x}\\ -c_{i,z} & c_{i,x} & 0 \end{bmatrix}, \quad for \quad s=3;$$

$$\mathbf{S}(\mathbf{c_i}) = \begin{bmatrix} -c_{i,y} & c_{i,x} \end{bmatrix}, \quad for \quad s=2;$$

blocks $\mathbf{D}_{i,j}$ and $\mathbf{L}_{i,j}$ are defined as

$$\mathbf{D}_{i,j} = \begin{cases} \mathbf{0}_{1\times s} & \text{if the } i\text{-th contact force does}\\ & \text{not affect the } j\text{-th joint;}\\ \mathbf{z}_j^T & \text{for prismatic } j\text{-th joint;}\\ \mathbf{z}_j'^T\ \mathbf{S}({}^\mathbf{f}\mathbf{c_i}-\mathbf{o_j}) & \text{for rotational } i\text{-th joint;} \end{cases}$$

$$\mathbf{L}_{i,j} = \begin{cases} \mathbf{0}_{1\times(d-s)} & \text{if the } i\text{-th contact force does}\\ & \text{not affect the } j\text{-th joint;}\\ \mathbf{0}_{1\times(d-s)} & \text{for prismatic } j\text{-th joint;}\\ \mathbf{z}_j'^T & \text{for rotational } j\text{-th joint;} \end{cases}$$

where $\mathbf{o}_j$ and $\mathbf{z}_j$ are the center and z-axis unit vector of the Denavit-Hartenberg frames associated with the j-th joint while $\mathbf{z}'_j = \mathbf{z}_j$ if $s = 3$ and $\mathbf{z}'_j = 1$ if $s = 2$.

In what follows the most common contact types are described with a unified notation, by means of an *overall contact selection matrix* $\mathbf{H}$ which is defined by removing the zero rows from the matrix

$$\tilde{\mathbf{H}} = \mathrm{diag}(FS_1, \ldots, FS_n, MS_1, \ldots, MS_n).$$

The force selector (FS_i) and the moment selector (MS_i) blocks are built according to different contact models [15, 8] as

Contact Type	Force Selector FS_i	Moment Selector MS_i
Point Contact w/o Friction	$\mathbf{n}_i^T$	$\mathbf{0}_{1\times(d-s)}$
Point Contact w/h Friction (Hard–Finger)	$\mathbf{I}_s$	$\mathbf{0}_{1\times(d-s)}$
Line Contact w/o Friction	$\mathbf{n}_i^T$	$(S(\mathbf{n}_i)\mathbf{x}_i)^T$
3D Line Contact w/h Friction	$\mathbf{n}_i^T$	$\begin{bmatrix} (S(\mathbf{n}_i)\mathbf{x}_i)^T \\ \mathbf{n}_i^T \end{bmatrix}$
3D Planar Contact w/o Friction	$\mathbf{n}_i^T$	$\begin{bmatrix} \mathbf{x}_i^T \\ \mathbf{y}_i^T \end{bmatrix}$
Planar Contact w/h Friction (Complete–Constraint)	$\mathbf{I}_s$	$\mathbf{I}_{d-s}$
3D Soft Finger	$\mathbf{I}_3$	$\mathbf{n}_i^T$

where $\mathbf{n}_i$ is the unit surface normal at the i–th contact point, $\mathbf{x}_i$ is a unit vectors defining the line of contact and ($\mathbf{x}_i$,$\mathbf{y}_i$) are two unit vectors defining the plane of contact. Notice that conventional kinematic joints between a link and the object can also be modelled. In fact, as for differential kinematic constraints, the following are equivalent:

Joint Type	Force Selector FS_i	Moment Selector MS_i
3D Rotoidal	$\mathbf{I}_3$	$\begin{bmatrix} \mathbf{x}_i^T \\ \mathbf{y}_i^T \end{bmatrix}$
2D Rotoidal	$\mathbf{I}_2$	0
3D Prismatic	$\begin{bmatrix} \mathbf{x}_i^T \\ \mathbf{y}_i^T \end{bmatrix}$	$\mathbf{I}_3$
2D Prismatic	$\mathbf{x}_i^T$	1
3D Spherical	$\mathbf{I}_3$	$\mathbf{0}_{1\times 3}$

where $\mathbf{x}_i$ and $\mathbf{y}_i$ denote two unit vectors normal to the joint axis $\mathbf{z}_i$.

In the robotic literature matrix

$$\mathbf{G} = \tilde{\mathbf{G}}\mathbf{H}^T$$

is usually termed as the "grasp matrix", or "grip transform", while

$$\mathbf{J} = \mathbf{H}\tilde{\mathbf{J}}$$

is referred to as the "hand Jacobian".

An important question in the differential kinematics analysis is: which object motions are possible starting from a given configuration, and to which joint motions do they correspond? This question can be easily answered if the mechanism under consideration is not "defective", i.e. if it has at least as any d.o.f. as necessary to achieve arbitrary configurations in its task space. In fact, in this case the matrix $\mathbf{J}$ is full rank, and we can write (1.5) as

$$\dot{\mathbf{q}} = \mathbf{J}^+\mathbf{G}^T\dot{\mathbf{u}} + \left(\mathbf{I} - \mathbf{J}^+\mathbf{J}\right)\mathbf{y}, \tag{1.6}$$

where $\mathbf{J}^+$ is the Moore-Penrose pseudo-inverse of $\mathbf{J}$, and $\mathbf{y}$ is a free vector that parameterizes the homogeneous (redundant) part of the solution. A manipulation system, however, generally contains defective kinematics members such as the inner links, and therefore $\mathbf{J}$ is not full rank. The relationship between $\dot{\mathbf{u}}$ and $\dot{\mathbf{q}}$ for general manipulation systems (including whole-arm) has been considered by Bicchi *et al.*, [4], where it was shown that there exist three vectors ν_1, ν_2, and ν_3 (whose dimensions vary with the problem at hand) such that every possible pair ($\dot{\mathbf{u}}$,$\dot{\mathbf{q}}$) of object and joint velocities that comply with the kinematic and contact constraints of the hand system can be written as

$$\dot{\mathbf{u}} = \Gamma_i\nu_1 + \Gamma_{uc}\nu_2; \tag{1.7}$$

$$\dot{\mathbf{q}} = \Gamma_{qc}\nu_2 + \Gamma_{qr}\nu_3. \tag{1.8}$$

The columns of Γ_{uc} and those of Γ_{qc} form a basis of the subspaces of coordinate object and joint velocities, respectively. Any object motion described by the coordinate vector ν_2 in the image of Γ_{uc} must correspond to a joint motion with the same coordinates in the basis Γ_{qc}. The images of Γ_{qr} and Γ_i represent the subspaces of redundant joint velocities and under-actuated object velocities, respectively. In the following the column space of matrix Γ_{uc} is referred to as the subspace of "rigid–body coordinated object motions".

1.3 Dynamics

The manipulation system consists of a constrained mechanical system, whose dynamical description can be derived using Lagrange's equations together with constraint equations. Consider first the dynamics of the hand and of the object separately:

$$\left(\frac{d}{dt}\frac{\partial L_h}{\partial \dot{\mathbf{q}}} - \frac{\partial L_h}{\partial \mathbf{q}}\right)^T = \mathbf{M}_h(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_h(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{V}_h(\mathbf{q}) = \boldsymbol{\tau} \tag{1.9}$$

$$\left(\frac{d}{dt}\frac{\partial L_o}{\partial \dot{\mathbf{u}}} - \frac{\partial L_o}{\partial \mathbf{u}}\right)^T = \mathbf{M}_o(\mathbf{u})\ddot{\mathbf{u}} + \mathbf{C}_o(\mathbf{u},\dot{\mathbf{u}})\dot{\mathbf{u}} + \mathbf{V}_o(\mathbf{u}) = \mathbf{w}, \tag{1.10}$$

where L_h and L_o are the manipulator and object Lagrangians, respectively, the $\mathbf{M}_i(\cdot)$ are inertia matrices, the $\mathbf{C}(\cdot,\cdot)$ terms include velocity-dependent forces, and the $\mathbf{V}(\cdot)$ terms represent gravity and friction forces. These two equations are then attached by means of the velocity constraint (1.5). Murray and Sastry [1990] discussed this dynamic problem in the hypothesis that the Jacobian $\mathbf{J}$ is full row rank, which fact allows to explicit the connected dynamics in terms of the independent variables $\mathbf{u}$ by using (1.6).

In general manipulation systems however, the hand Jacobian may not be full–rank. By introducing the undetermined t–dimensional vector of Lagrange multipliers $\mathbf{t}$, the virtual work of the connected system can be written as

$$\left[\frac{d}{dt}\frac{\partial(L_h+L_o)}{\partial(\dot{\mathbf{q}},\dot{\mathbf{u}})} - \frac{\partial(L_h+L_o)}{\partial(\mathbf{q},\mathbf{u})} + \mathbf{t}^T\left[\mathbf{J} - \mathbf{G}^T\right] - (\boldsymbol{\tau}^T\ \mathbf{w}^T)\right]\begin{bmatrix}\delta\mathbf{q}\\ \delta\mathbf{u}\end{bmatrix} = \mathbf{0},$$

whence, differentiating (1.5) to eliminate virtual displacements, one gets

$$\mathbf{M}_{dyn}\begin{bmatrix}\ddot{\mathbf{q}}\\ \ddot{\mathbf{u}}\\ \mathbf{t}\end{bmatrix} = \begin{bmatrix}\tau - \mathbf{Q}_h\\ \mathbf{w} - \mathbf{Q}_o\\ \mathbf{Q}_c\end{bmatrix}, \tag{1.11}$$

where

$$\mathbf{M}_{dyn} = \begin{bmatrix}\mathbf{M}_h & \mathbf{0} & \mathbf{J}^T\\ \mathbf{0} & \mathbf{M}_o & -\mathbf{G}\\ \mathbf{J} & -\mathbf{G}^T & \mathbf{0}\end{bmatrix}, \tag{1.12}$$

and $\mathbf{Q}_c = \frac{\partial \mathbf{J}\dot{\mathbf{q}}}{\partial \mathbf{q}}\dot{\mathbf{q}} - \frac{\partial \mathbf{G}^T\dot{\mathbf{u}}}{\partial \mathbf{u}}\dot{\mathbf{u}}$.

With respect to the structure of the Jacobian and grasp matrices, some relevant characteristics of the manipulation system are summarized in the following definition. Here $\mathsf{ker}\,(\mathbf{Q})$ denotes the kernel (or right nullspace) of matrix $\mathbf{Q}$:

Definition 1 A manipulation system is said "defective" if $\mathsf{ker}\,(\mathbf{J}^T) \neq \mathbf{0}$; "(motion) indeterminate" if $\mathsf{ker}\,(\mathbf{G}^T) \neq \mathbf{0}$; "redundant" if $\mathsf{ker}\,(\mathbf{J}) \neq \mathbf{0}$; "graspable" if $\mathsf{ker}\,(\mathbf{G}) \neq \mathbf{0}$ and "hyperstatic" if $\mathsf{ker}\,(\mathbf{J}^T) \cap \mathsf{ker}\,(\mathbf{G}) \neq \mathbf{0}$.

Remark 1 The term "defective" is employed because the row rank of the Jacobian is not full when at least one of the links touching the object possesses less degrees–of–freedom than those necessary to move its contact point in all directions inhibited by the relative contact constraint. Equivalently, in terms of forces, defectivity implies that there exists at least one direction of the contact force $\mathbf{t}$ that does not affect the manipulator joint torques. Defectivity occurs whenever the number t of components of contact forces is larger than the number q of joints, or when the manipulator is in a singular configuration.

The term "motion indeterminate", or "indeterminate" for short, refers to the fact that the object is not completely restrained by contacts, and hence its motion can not be determined quasi–statically (indeterminacy of motion is of course solved when dynamics are taken into account).

The term "redundant" is standard in robotics. Note that here, redundancy of one of the linkages is enough to have redundancy of the whole system, and that redundancy and defectivity may occur in the same mechanism.

The term “graspable” refers to the fact that self–balanced “squeezing” contact forces are possible in this case, so that a multi–finger frictional grasp is possible.

Finally, we use “hyperstatic” for systems where the distribution of contact forces can not be determined by knowledge of joint torques and external forces alone. Such systems have also been termed “indeterminate” with reference to force, but we prefer to avoid this usage here because of possible confusion with motion indeterminacy. •

Figure 1.1 pictorially illustrates such definitions. The class of “general manipulation systems”

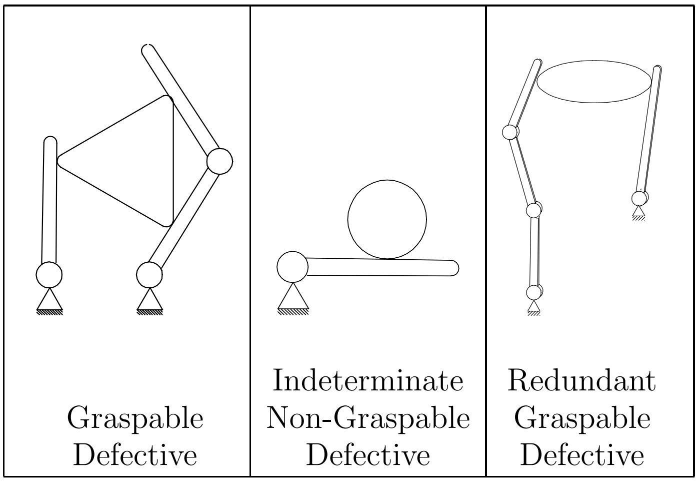

Figure 1.1 Illustration of mechanism characteristics.

this chapter is concerned with is comprised of mechanisms with any number of limbs (open kinematic chains), of joints (prismatic, rotoidal, spherical, etc.) and of contacts (hard and soft finger, complete–constraint, etc.) between a reference member called “object” and links in any position in the limb chains. This includes in particular defective and hyperstatic systems, whose treatment is seldom considered in the literature.

By observing that

$$\ker \mathbf{M}_{dyn} = \{(\ddot{\mathbf{q}},\ddot{\mathbf{u}},\mathbf{t})^T |\ \ddot{\mathbf{q}} = \mathbf{0},\ \ddot{\mathbf{u}} = \mathbf{0},\ \mathbf{t} \in \ker \mathbf{J}^T \cap \ker \mathbf{G}\},$$

it ensues that, for all hyperstatic grasps, the matrix $\mathbf{M}_{dyn}$ of the rigid–body dynamics in (1.11) is not invertible and the law of motion of the manipulation system results indeterminate.

Figure 1.3 pictorially describes the notions of defectivity, graspability and hyperstaticity for two simple manipulator systems.

Due to the generality of systems under consideration, rigid–body models are not satisfactory. In fact, many interesting manipulation systems are indeed hyperstatic, as in the case of whole–arm robots. Moreover rigid–body dynamics do not allow proper modelization, and hence control, of contact forces (closed–loop control of forces would in fact entail algebraic loops). Because contact force control is a central point in grasping, this is certainly an important drawback of the rigid body dynamics approach. Finally, systems with significant

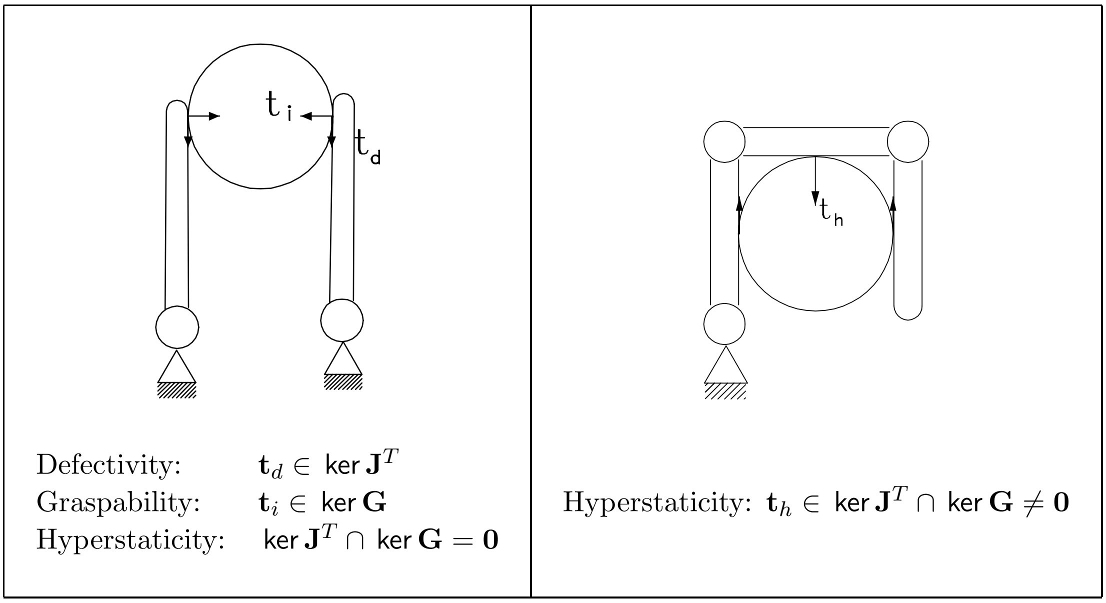

Figure 1.2 Examples of defective, graspable and hyperstatic grasps.

inherent compliance are sometimes encountered, especially in applications where stable and accurate force control is of concern.

To address such more general cases, it is necessary to introduce further structure in the mechanical model, namely, elastic energy terms

$$K_i = \frac{1}{2}\xi_i^T({}^o\mathbf{c}_i, {}^h\mathbf{c}_i)\mathbf{K}_i\xi_i({}^o\mathbf{c}_i, {}^h\mathbf{c}_i),$$

and dissipation terms

$$B_i = \frac{1}{2}\dot{\xi}_i^T({}^o\mathbf{c}_i, {}^h\mathbf{c}_i, {}^o\dot{\mathbf{c}}_i, {}^h\dot{\mathbf{c}}_i)\mathbf{B}_i\dot{\xi}_i({}^o\mathbf{c}_i, {}^h\mathbf{c}_i, {}^o\dot{\mathbf{c}}_i, {}^h\dot{\mathbf{c}}_i),$$

where $\mathbf{K}_i$, $\mathbf{B}_i$ are symmetric, positive definite matrices incorporating (hand/object) material "stiffness" and "damping" characteristics, and $\xi_i(\cdot,\cdot)$ is a suitable displacement function[2] applied to the position of the reference frames on the object and finger surfaces at the i–th contact point.

Having included the elastic energy and dissipation terms in the model of the whole system, the standard derivation of the now decoupled dynamics can now be applied and gives

$$\mathbf{M}_h\ddot{\mathbf{q}} + \mathbf{Q}_h + \left[\frac{\partial\xi}{\partial^o\mathbf{c}}\frac{\partial^o\mathbf{c}}{\partial\mathbf{q}} + \frac{\partial\xi}{\partial^h\mathbf{c}}\frac{\partial^h\mathbf{c}}{\partial\mathbf{q}}\right]^T\mathbf{K}\xi + \left[\frac{\partial\dot{\xi}}{\partial^o\dot{\mathbf{c}}}\frac{\partial^o\dot{\mathbf{c}}}{\partial\dot{\mathbf{q}}} + \frac{\partial\dot{\xi}}{\partial^h\dot{\mathbf{c}}}\frac{\partial^h\dot{\mathbf{c}}}{\partial\dot{\mathbf{q}}}\right]^T\mathbf{B}\dot{\xi} = \tau; \qquad (1.13)$$

$$\mathbf{M}_o\ddot{\mathbf{u}} + \mathbf{Q}_o + \left[\frac{\partial\xi}{\partial^o\mathbf{c}}\frac{\partial^o\mathbf{c}}{\partial\mathbf{u}} + \frac{\partial\xi}{\partial^h\mathbf{c}}\frac{\partial^h\mathbf{c}}{\partial\mathbf{u}}\right]^T\mathbf{K}\xi + \left[\frac{\partial\dot{\xi}}{\partial^o\dot{\mathbf{c}}}\frac{\partial^o\dot{\mathbf{c}}}{\partial\dot{\mathbf{u}}} + \frac{\partial\dot{\xi}}{\partial^h\dot{\mathbf{c}}}\frac{\partial^h\dot{\mathbf{c}}}{\partial\dot{\mathbf{u}}}\right]^T\mathbf{B}\dot{\xi} = \mathbf{w}, \qquad (1.14)$$

[2]The proper choice of this displacement function is actually an hard problem in the analysis of contact mechanics, see e.g. [7]. A detailed discussion of this point may be found in [16].

where **K** and **B** are the aggregated stiffness and damping matrices for the manipulation system. Computation of these matrices based on knowledge of visco–elastic parameters of contacting bodies is possible along the lines of [5]. Although in practice such knowledge might be difficult to obtain, procedures similar to those currently used to identify inertial parameters of robot arms can be conceivably used to estimate visco–elastic parameters. Notice that the lumped–parameter model for visco–elastic interactions strongly simplifies the (possibly on–line) identification procedure.

The following assumptions are introduced:

A1 $\xi({}^h\mathbf{c},{}^o\mathbf{c}) = \mathbf{H}({}^h\mathbf{c} -{}^o \mathbf{c})$. This amounts to assuming a linear elastic model for the bodies.

A2 Contact points do not change by rolling. From the identity

$$\frac{\partial {}^o\dot{\mathbf{c}}}{\partial \dot{\mathbf{u}}} = \frac{\partial}{\partial \dot{\mathbf{u}}}\left(\frac{\partial {}^o\mathbf{c}}{\partial t} + \frac{\partial {}^o\mathbf{c}}{\partial \mathbf{u}}\,\dot{\mathbf{u}}\right) = \frac{\partial {}^o\mathbf{c}}{\partial \mathbf{u}},$$

one gets $\frac{\partial {}^o\mathbf{c}}{\partial \mathbf{u}} = \tilde{\mathbf{G}}^T(\mathbf{u})$. Similarly, $\frac{\partial {}^h\mathbf{c}}{\partial \mathbf{q}} = \tilde{\mathbf{J}}(\mathbf{q})$. Further, $\frac{\partial {}^h\mathbf{c}}{\partial \mathbf{u}} = \frac{\partial {}^o\mathbf{c}}{\partial \mathbf{q}} = \mathbf{0}$. Non–rolling contacts can be reasonably assumed when the relative curvature of the contacting bodies is high.

In this setting, the Lagrange multipliers $\mathbf{t}$ can be interpreted as representing the vector of constraint forces deriving from virtual "springs" and "dampers" with endpoints attached at the contact points ${}^o\mathbf{c}_i$'s and ${}^h\mathbf{c}_i$'s, as

$$\mathbf{t} = \mathbf{KH}({}^h\mathbf{c} -{}^o \mathbf{c}) + \mathbf{BH}({}^h\dot{\mathbf{c}} -{}^o \dot{\mathbf{c}}). \tag{1.15}$$

Accordingly, (1.13) and (1.14) can be rewritten as

$$\ddot{\mathbf{q}} = \mathbf{M}_h^{-1}\left(-\mathbf{Q}_h - \mathbf{J}^T\mathbf{t} + \tau\right); \tag{1.16}$$

$$\ddot{\mathbf{u}} = \mathbf{M}_o^{-1}\left(-\mathbf{Q}_o + \mathbf{G}\mathbf{t} + \mathbf{w}\right). \tag{1.17}$$

1.3.1 Linearization

For the analysis of most of the structural properties of general manipulation systems, the model (1.16)–(1.17) is still intractable. Henceforth, then, we will deal with the linearized dynamic model

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_\tau\tau' + \mathbf{B}_w\mathbf{w}', \tag{1.18}$$

where the state vector $\mathbf{x} \in \mathbb{R}^{2(q+d)}$, inputs $\tau' \in \mathbb{R}^q$, and disturbances $\mathbf{w}' \in \mathbb{R}^d$ are defined as the departures from a reference equilibrium configuration $\mathbf{x}_{eq} = [\mathbf{q}_{eq}^T\ \mathbf{u}_{eq}^T\ \mathbf{0}\ \mathbf{0}]^T$ at which contact forces are $\mathbf{t}(\mathbf{x}_{eq}) = \mathbf{t}_{eq}$, as

$$\begin{aligned}
\mathbf{x} &= \left[\delta\mathbf{q}^T\ \delta\mathbf{u}^T\ \dot{\mathbf{q}}^T\ \dot{\mathbf{u}}^T\right]^T = \left[(\mathbf{q}-\mathbf{q}_{eq})^T\ (\mathbf{u}-\mathbf{u}_{eq})^T\ \dot{\mathbf{q}}^T\ \dot{\mathbf{u}}^T\right]^T, \\
\tau' &= \tau - \mathbf{J}^T\mathbf{t}_{eq}, \\
\mathbf{w}' &= \mathbf{w} + \mathbf{G}\mathbf{t}_{eq}.
\end{aligned} \tag{1.19}$$

The dynamics matrix $\mathbf{A}$, joint torque input matrix $\mathbf{B}_\tau$, and external wrench disturbance matrix $\mathbf{B}_w$ have the form

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{L}_k & -\mathbf{L}_b \end{bmatrix}; \quad \mathbf{B}_\tau = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_h^{-1} \\ \mathbf{0} \end{bmatrix}; \quad \mathbf{B}_w = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}_o^{-1} \end{bmatrix}, \tag{1.20}$$

where

$$\mathbf{L}_k = -\begin{bmatrix} \mathbf{M}_h^{-1}\left(-\frac{\partial(\mathbf{v}_h - \mathbf{J}^T\mathbf{t}_{eq})}{\partial \mathbf{q}} - \mathbf{J}^T\mathbf{K}\mathbf{J}\right) & \mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{K}\mathbf{G}^T \\ \mathbf{M}_o^{-1}\mathbf{G}\mathbf{K}\mathbf{J} & \mathbf{M}_o^{-1}\left(\frac{\partial(\mathbf{v}_o + \mathbf{G}\mathbf{t}_{eq})}{\partial \mathbf{u}} - \mathbf{G}\mathbf{K}\mathbf{G}^T\right) \end{bmatrix};$$

$$\mathbf{L}_b = -\begin{bmatrix} -\mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{B}\mathbf{J} & \mathbf{M}_h^{-1}\mathbf{J}^T\mathbf{B}\mathbf{G}^T \\ \mathbf{M}_o^{-1}\mathbf{G}\mathbf{B}\mathbf{J} & -\mathbf{M}_o^{-1}\mathbf{G}\mathbf{B}\mathbf{G}^T \end{bmatrix},$$

and $\mathbf{v}_h$ and $\mathbf{v}_o$ denote the gravitational parts of $\mathbf{Q}_h$ and $\mathbf{Q}_o$, respectively. All the matrices of the linearized dynamic model are implicitly assumed to be evaluated at the equilibrium configuration. Notice that, in case some of the joints are not actuated (as it may happen in considering closed kinematic chains), the corresponding columns of $\mathbf{B}_tau$ need to be deleted.

In the general case, block $\mathbf{L}_k$ still has a rather involved expression in terms of the system's kinematic parameters and material properties, and depends on the intensity of forces at equilibrium. To the purpose of obtaining clearly intelligible results relating structural properties of manipulation systems to their more intrinsic parameters, henceforth the linearized model is considered under further assumptions as follows:

A3 Terms due to gravity variation $\frac{\partial \mathbf{v}_h}{\partial \mathbf{q}}$ and $\frac{\partial \mathbf{v}_o}{\partial \mathbf{u}}$ are negligible.

A4 Stiffness and damping matrices are proportional $\mathbf{B} \propto \mathbf{K}$. Note that this is a customary assumption in mechanical vibration analysis (see e.g. [9]).

A5 $\mathbf{J}(\mathbf{q})$ and $\mathbf{G}(\mathbf{u})$ are slowly varying functions of their arguments, so that terms $\frac{\partial \mathbf{J}^T\mathbf{t}_{eq}}{\partial \mathbf{q}}$, $\frac{\partial \mathbf{G}\mathbf{t}_{eq}}{\partial \mathbf{u}}$ are negligible. Note that assuming small contact forces at the equilibrium has the same effect on the linearizing approximation.

Under these assumptions, we have

$$\mathbf{L}_k = \mathbf{M}^{-1}\mathbf{P}_k; \qquad \mathbf{L}_b = \mathbf{M}^{-1}\mathbf{P}_b, \tag{1.21}$$

where

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_h & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_o \end{bmatrix}; \quad \mathbf{P}_k = \begin{bmatrix} \mathbf{J}^T \\ -\mathbf{G} \end{bmatrix} \mathbf{K} \begin{bmatrix} \mathbf{J} & -\mathbf{G}^T \end{bmatrix}; \quad \mathbf{P}_b = \begin{bmatrix} \mathbf{J}^T \\ -\mathbf{G} \end{bmatrix} \mathbf{B} \begin{bmatrix} \mathbf{J} & -\mathbf{G}^T \end{bmatrix}. \tag{1.22}$$

As the goal of dextrous manipulation is to control the position of the manipulated object through the contact forces exerted by the fingers, it is natural to take the object and limbs position and the contact forces as outputs of our system. For the linearized model under consideration, from (1.19) and (1.15), it can be written

$$\delta\mathbf{u} = \mathbf{C}_u\mathbf{x}, \quad \text{with} \quad \mathbf{C}_u = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}; \tag{1.23}$$

$$\delta \mathbf{q} = \mathbf{C}_q \mathbf{x}, \quad \text{with} \quad \mathbf{C}_q = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}; \tag{1.24}$$

$$\delta \mathbf{t} = \mathbf{C}_t \mathbf{x}, \quad \text{with} \quad \mathbf{C}_t = \begin{bmatrix} \mathbf{KJ} & -\mathbf{KG}^T & \mathbf{BJ} & -\mathbf{BG}^T \end{bmatrix}. \tag{1.25}$$

In [12] the structural properties of pointwise controllability and observability from object and joint positions and from contact forces, were analyzed for manipulation systems with general kinematics. The results are summarized in a standard form of the dynamics equations which is reported in the following theorem.

Theorem 1 *For a general manipulation system there always exists a change of coordinates* $\mathbf{z} = \hat{\mathbf{T}}^{-1}\mathbf{x}$*, such that the linearized model (1.18) takes on the form*

$$\hat{\mathbf{T}}^{-1}\mathbf{A}\hat{\mathbf{T}} = \left[\begin{array}{c|c|c|c|c|c} {}^{r}\mathbf{A} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & {}^{h}\mathbf{A} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & {}^{c}\mathbf{A} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & {}^{a}\mathbf{A} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & {}^{i}\mathbf{A} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & {}^{d}\mathbf{A} \end{array}\right]; \quad \hat{\mathbf{T}}^{-1}\mathbf{B}_\tau = \left[\begin{array}{c} \bullet \\ \hline \bullet \\ \hline \bullet \\ \hline \bullet \\ \hline \mathbf{0} \\ \hline \mathbf{0} \end{array}\right]; \quad \hat{\mathbf{T}}^{-1}\mathbf{B}_w = \left[\begin{array}{c} \mathbf{0} \\ \hline \mathbf{0} \\ \hline \bullet \\ \hline \bullet \\ \hline \bullet \\ \hline \bullet \end{array}\right];$$

$$\mathbf{C}_u\hat{\mathbf{T}} = \left[\begin{array}{c|c|c|c|c|c} \mathbf{0} & \mathbf{0} & \bullet & \bullet & \bullet & \bullet \end{array}\right];$$

$$\mathbf{C}_q\hat{\mathbf{T}} = \left[\begin{array}{c|c|c|c|c|c} \bullet & \bullet & \bullet & \bullet & \mathbf{0} & \mathbf{0} \end{array}\right];$$

$$\mathbf{C}_t\hat{\mathbf{T}} = \left[\begin{array}{c|c|c|c|c|c} \mathbf{0} & \bullet & \mathbf{0} & \bullet & \mathbf{0} & \bullet \end{array}\right].$$

The dynamics corresponding to the diagonal partitions of the state matrix are those of "redundant", "identically internal", "coordinate", "active", "indeterminate" and "defective" subsystems.

Such form synthetically contains information relating to the structural properties of the various subsystems. It can be seen, for instance, that the free evolution of the system from non–zero initial conditions belonging to any one of the fundamental subspaces (redundant, dynamically internal, core, indeterminate, and defective), remains inside the same subspace. In other words, the fundamental modes are dynamically decoupled and can be independently excited.

Furthermore, as it can be easily recovered from application of the Popov–Belevich–Hautus (P.B.H.) lemma test, the lack of one of the five properties considered (controllability from joint torques and from disturbances, observability from object displacements, from joint displacements, and from contact forces) for a particular subsystem is indicated by the presence of a zero block in the corresponding position of the input or output matrices.

1.4 Output specification and controllability

In this section the specification of the controlled outputs and their controllability are analysed. The pointwise–controllable output subspace for contact forces can be evaluated [12] as

$$\mathbf{C}_t \min \mathcal{I}\left(\mathbf{A}, \mathsf{im}\left(\mathbf{B}_\tau\right)\right) = \min \mathcal{I}\left(\Lambda, \mathsf{im}\left(\mathbf{KJ}\right)\right),$$

where $\Lambda = -\mathbf{K}(\mathbf{J}{\mathbf{M}_h}^{-1}\mathbf{J}^T + \mathbf{G}^T{\mathbf{M}_o}^{-1}\mathbf{G})$, and $\min \mathcal{I}\left(\mathbf{A}, \mathsf{im}\left(\mathbf{B}_\tau\right)\right) = \sum_{i=0}^{n-1} \mathbf{A}^i \, \mathsf{im}\left(\mathbf{B}_\tau\right)$ is the minimum $\mathbf{A}$–invariant subspace containing $\mathsf{im}\left(\mathbf{B}_\tau\right)$.

A particularly important concern in robotic manipulation is to avoid slippage at the contacts by controlling *internal forces.* These forces are self–balanced contact forces which do not influence the object dynamics. From (1.17) it follows that the subspace of *internal forces* corresponds to the nullspace of the grasp matrix $\mathbf{G}$. Although the notion of *internal forces* is strictly related to the object grasp configuration, their controllability strongly depends on the kinematics and on the actuation of the grasping mechanism. In fact, the *subspace of controllable internal forces*, i.e. the intersection of the set of controllable forces with that of internal forces,

$$\mathcal{F}_{hr} = \mathbf{C}_t \min \mathcal{I}\,(\mathbf{A}, \mathrm{im}\,\mathbf{B}_\tau) \cap \ker \mathbf{G}.$$

is in general only a proper subset of both subspaces.

Bicchi [3] and Prattichizzo *et al.* [14] proved (in a quasi–static and dynamic setting, respectively) the geometric description of the controllable internal force subspaces to be given as

$$\mathcal{F}_{hr} = \mathrm{im}\,(\mathbf{I} - \mathbf{K}\mathbf{G}^T(\mathbf{G}\mathbf{K}\mathbf{G}^T)^{-1}\mathbf{G})\mathbf{K}\mathbf{J}.$$

The following proposition (whose proof is omitted) highlights the close tie between the hyperstaticity and the loss of internal force controllability.

Proposition 1 *The subspace of reachable internal forces* $\mathcal{F}_{hr}$ *is equal to the subspace of internal forces* $\ker \mathbf{G}$ *if and only if the manipulation system is not hyperstatic, i.e.* $\ker(\mathbf{J}^T) \cap \ker(\mathbf{G}) = \{0\}$.

As regards object motions, it holds

$$\mathbf{C}_u \min \mathcal{I}\,(\mathbf{A}, \mathrm{im}\,\mathbf{B}_\tau) = \mathbf{M}_o^{-1}\mathbf{G}\ (\mathbf{C}_t \min \mathcal{I}\,(\mathbf{A}, \mathrm{im}\,\mathbf{B}_\tau)) = \mathbf{M}_o^{-1}\mathbf{G} \min \mathcal{I}\,(\Lambda, \mathrm{im}\,(\mathbf{K}\mathbf{J})).$$

Notice that arbitrary object positions can be reached if and only if the grasp map $\mathbf{G}$ is onto and the force controllability map $\mathbf{C}_t \min \mathcal{I}\,(\mathbf{A}, \mathrm{im}\,(\mathbf{B}_\tau)$ is injective on $\mathrm{im}\,(\mathbf{G}^T)$. More specifically, particular attention should be paid to the subspace of coordinated object motions $\mathrm{im}\,(\Gamma_{uc})$ defined in (1.8). This subspace is related to the rigid–body kinematics which are of particular interest in the control of manipulation systems. Since they do not involve visco–elastic deformations of bodies, they can be regarded as low–energy motions. In a sense, they represent the natural way to change the object posture. In [13] authors proved that the subspace of coordinated object motions is controllable, i.e.

$$\mathrm{im}\,(\Gamma_{uc}) \subset \mathbf{C}_u \min \mathcal{I}\,(\mathbf{A}, \mathrm{im}\,\mathbf{B}_\tau).$$

According to the previous discussion, not all object motions and contact forces result controllable by joint torques in manipulation systems with general kinematics. In order to allow correct specification of a manipulation task, it is important to clearly understand which are the motions and forces that can be exactly controlled. To do this, define three outputs as

$$\mathbf{e}_{uc} = \mathbf{E}_{uc}\mathbf{x} \stackrel{def}{=} \Gamma_{uc}^{+}\mathbf{C}_u\mathbf{x}; \tag{1.26}$$

$$\mathbf{e}_{ti} = \mathbf{E}_{ti}\mathbf{x} \stackrel{def}{=} \mathbf{E}^{+}\mathbf{C}_t\mathbf{x}; \tag{1.27}$$

$$\mathbf{e}_{qr} = \mathbf{E}_{qr}\mathbf{x} \stackrel{def}{=} \begin{bmatrix} \mathbf{\Gamma}_r^{+}\mathbf{M}_h & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{x}, \tag{1.28}$$

where $\mathbf{\Gamma}_r$ and $\mathbf{E}$ are basis matrices for $\ker(\mathbf{J})$ and $\mathcal{F}_{hr}$, respectively, while $\mathrm{im}(\Gamma_{uc})$ is the subspace of the coordinate object motions (1.8). The set of the above three output vectors, grouped in the output vector

$$\mathbf{e} = \begin{bmatrix} \mathbf{e}_{uc} \\ \mathbf{e}_{ti} \\ \mathbf{e}_{qr} \end{bmatrix} = \mathbf{C}\mathbf{x} = \begin{bmatrix} \mathbf{\Gamma}_{uc}^{+}\mathbf{C}_{\mathbf{u}} \\ \mathbf{E}^{+}\mathbf{C}_{\mathbf{t}} \\ \mathbf{\Gamma}_{r}^{+}\mathbf{C}_{\mathbf{q}} \end{bmatrix}, \tag{1.29}$$

is guaranteed to be pointwise–controllable (see [13]), and is convenient for the specification of the manipulation tasks, i.e. it incorporates the typical subtasks of a manipulation task with their priorities:

a) object trajectories which can be accommodated for by the mechanism;
b) contact forces which can be steered so as to avoid violation of contact constraints;
c) reconfiguration of limbs in presence of redundancy.

1.5 Force/motion functional controllability

The capability of following a desired trajectory with the manipulated object, while guaranteeing that contact forces are controlled so as to comply with contact constraints at every instant, is not guaranteed by pointwise–controllability alone. In system theory this problem is known as "functional (output trajectory) controllability". Although functional controllability is generally approached by state–space methods (Sain and Massey, 1969), for linear systems it is most simply studied in terms of input–output representations. A well–known necessary and sufficient condition for the output functional controllability of linear system is that its transfer function matrix is full row rank over the field of complex numbers (notice that output functional controllability requires that at least as many inputs are available as there are outputs of concern).

In this section we show that the set of outputs defined in (1.29) is functionally controllable. In order to do this, the concept of "asymptotic reproducibility" (Brocket and Mesarovich, 1965) is well suited. Asymptotic reproducibility investigates output tracking for a particular class of trajectories, namely those constant in time. In other words, it investigates the possibility of displacing the system from its reference equilibrium configuration to a different nearby equilibrium by means of step inputs. Note that the asymptotic reproducibility of the outputs of an asymptotically stable system is a sufficient condition for the functional reproducibility of the same outputs. The following theorem, proven in [13], shows that the task–oriented output vector (1.29) enjoys the property of being functionally controllable:

Theorem 2 *In the hypothesis that* $\ker(\mathbf{G}^T) = \mathbf{0}$*, consider the linearized dynamics described by the triple* $(\mathbf{A},\mathbf{B}_\tau,\mathbf{C})$*, where* $\mathbf{A}$*,* $\mathbf{B}_\tau$ *and* $\mathbf{C}$ *are as in Section 1.3.1 and in (1.29). Then for any constant linear state feedback* $\mathbf{R} = [\mathbf{R}_q\ \mathbf{R}_u\ \mathbf{R}_{\dot{q}}\ \mathbf{R}_{\dot{u}}]$ *such that* $\mathbf{A} - \mathbf{B}_\tau\mathbf{R}$ *is asymptotically stable, the system* $(\mathbf{A} - \mathbf{B}_\tau\mathbf{R},\ \mathbf{B}_\tau,\ \mathbf{C})$ *is square and functionally controllable.*

The controlled output vector (1.29) consisting of coordinates for the subspace of rigid–body displacements of the manipulated object, of active internal contact forces and of redundant joints' motions provides a basis of the set of all functionally controllable outputs, that exactly corresponds to the task specifications introduced above and exhausts the control capabilities of the manipulation system.

The practical relevance of this proposition is that in [13] authors proved that independent proportional-derivative control at joints, i.e. $\mathbf{R} = [\mathbf{R}_q \, \mathbf{0} \, \mathbf{R}_{\dot{\mathbf{q}}} \, \mathbf{0}]$, is sufficient to stabilize any manipulation system whose motions are quasi–statically determinate, about a reference equilibrium.

1.6 Force/motion decoupling

As a direct application of Theorem 2 let us consider the design of a steady–state decoupling prefilter for manipulation systems with general kinematics. Being $\mathbf{W}(s)$ the transfer function of the triple $(\mathbf{A} - \mathbf{B}_\tau \mathbf{R},\ \mathbf{B}_\tau,\ \mathbf{C})$, a steady–state input/output decoupling prefilter for the input–output representation $\mathbf{y}(s) = \mathbf{W}(s)\tau(s)$ can be simply obtained as

$$\tau(s) = \mathbf{W}(0)^{-1}\nu(s).$$

Through this prefilter, reference steady–state values of object positions, contact forces and redundant variables are independently commanded by steps in $\nu_1(s)$, $\nu_2(s)$ and $\nu_3(s)$, respectively.

While the above simple open–loop filter achieves steady–state decoupling and potentially simplifies the design of the independent control loops, a more ambitious control goal concerns perfect input–output decoupling. While in principle perfect decoupling could be achieved (given the functional controllability of outputs) in open loop by using the inverse of the system's transfer function matrix, this is clearly not practically feasible, due to non–causality problems. In the following we will consider the perfect decoupling of the controlled outputs by means of state feedback. More in detail, we will show that noninteraction, by state feedback, of the rigid–body object motions, the reachable internal forces and of the manipulator redundancy is a structural property of manipulation systems with general kinematics, excluding only the case of systems with indeterminate motions. The geometric approach is used in such analysis. In particular, the result of this section regards the noninteracting control of general manipulation mechanisms and is based on necessary and sufficient conditions for the existence of the noninteraction control law given in [1].

Definition 2 *A control law for the dynamic system (1.18) is* noninteracting *with respect to the regulated outputs* $\mathbf{e}_{uc}$,$\mathbf{e}_{ti}$ *and* $\mathbf{e}_{qr}$ *if there exists a partition* τ_{uc},τ_{ti} *and* τ_{qr} *of the input vector* τ *such that for zero initial condition, each input* $\tau_{(\cdot)}$ *(with all other inputs identically zero) only affects the corresponding output* $e_{(\cdot)}$.

The following theorem, proved in [14] states that the noninteraction of the regulated outputs $\mathbf{e}_{uc}$,$\mathbf{e}_{ti}$ and $\mathbf{e}_{qr}$ for the dynamic system (1.18), is an intrinsic structural property of general manipulation systems.

Theorem 3 (Noninteraction) *Consider the linearized dynamics (1.18) of a manipulation system. Under the hypothesis that the system is not indeterminate* $(\ker(\mathbf{G}^T) = \{0\})$, *there exists a state–feedback matrix* $\mathbf{F}$ *such that the outputs*

- ☐ *the rigid–body object motions* $\mathbf{e}_{uc}$*;*

□ *the reachable internal forces* $\mathbf{e}_{ti}$*;*

□ *the mechanism redundancy* $\mathbf{e}_{qr}$*;*

are noninteracting.

Remark 2 In Theorem 2 we proved that the outputs, $\mathbf{e}_{uc}$, $\mathbf{e}_{ti}$ and $\mathbf{e}_{qr}$, are functionally controllable and exhaust the control capabilities, i.e. the input–output representation is invertible and square. In the last theorem we prove that there always exists a state–feedback controller which is *noninteracting* with respect to these outputs. •

The geometric concept from which the previous result develops is the $\mathcal{S}$*–constrained controllability*. It consists of those state space vectors reachable through trajectories entirely lying in the constraining subspace $\mathcal{S}$.

In other words, for the aforementioned outputs $\mathbf{e}_{ti}$ and $\mathbf{e}_{uc}$, there exists a decoupling and stabilizing state feedback matrix $\mathbf{F}$, along with two input partition matrices $\mathbf{U}_{ti}$ and $\mathbf{U}_{uc}$ such that, for the dynamic triples

$$\begin{gathered}(\mathbf{E}_{ti}, \mathbf{A}+\mathbf{B}_\tau\mathbf{F}, \mathbf{B}_\tau\mathbf{U}_{ti});\\(\mathbf{E}_{uc}, \mathbf{A}+\mathbf{B}_\tau\mathbf{F}, \mathbf{B}_\tau\mathbf{U}_{uc}),\end{gathered} \tag{1.30}$$

it holds:

$$\begin{aligned}\mathcal{R}_{ti} &= \min\mathcal{I}(\mathbf{A}+\mathbf{B}_\tau\mathbf{F}, \mathbf{B}_\tau\mathbf{U}_{ti}) \subseteq \ker(\mathbf{E}_{uc});\\ \mathbf{E}_{ti}\mathcal{R}_{ti} &= \mathrm{im}(\mathbf{E}_{ti});\\ \mathcal{R}_{uc} &= \min\mathcal{I}(\mathbf{A}+\mathbf{B}_\tau\mathbf{F}, \mathbf{B}_\tau\mathbf{U}_{uc}) \subseteq \ker(\mathbf{E}_{ti});\\ \mathbf{E}_{uc}\mathcal{R}_{uc} &= \mathrm{im}(\mathbf{E}_{uc}).\end{aligned} \tag{1.31}$$

Here, $\max\mathcal{I}(\mathbf{A}, \ker(C)) = \bigcap_{i=0}^{n-1}\mathbf{A}^i\ker(C)$ is the maximum $\mathbf{A}$–invariant subspace contained in $\ker(\mathbf{C})$ with respect to the triple ($\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$).

Moreover, matrices $\mathbf{U}_{uc}$ and $\mathbf{U}_{ti}$ satisfy the following relationships

$$\begin{aligned}\mathrm{im}(\mathbf{B}_\tau\mathbf{U}_{uc}) &= \mathrm{im}(\mathbf{B}_\tau)\cap\mathcal{R}_{uc};\\ \mathrm{im}(\mathbf{B}_\tau\mathbf{U}_{ti}) &= \mathrm{im}(\mathbf{B}_\tau)\cap\mathcal{R}_{ti}\end{aligned} \tag{1.32}$$

and the stabilizing matrix $\mathbf{F}$ is such that

$$\begin{aligned}(\mathbf{A}+\mathbf{B}_\tau\mathbf{F})\mathcal{R}_{uc} &\subseteq \mathcal{R}_{uc};\\ (\mathbf{A}+\mathbf{B}_\tau\mathbf{F})\mathcal{R}_{ti} &\subseteq \mathcal{R}_{ti}.\end{aligned} \tag{1.33}$$

1.7 Conclusions

In this paper we considered a linearized model of rather general mechanical systems for manipulation, and discussed in some detail results available from the literature on their structural properties and geometric control. Being robotic systems highly nonlinear in nature, one may question the validity of the linearization approach to the analysis. As a justification of this approach, the simplicity of results achievable by linearization appeared to be important at this rather early stage of investigation of complex manipulation systems. Moreover, it is well known that some of the results on the linearized system imply analogous local properties for the full system. It can be noted that conditions on the linearized system are only sufficient in general, and that wider applicability of some property may hold for the nonlinear system. This is the case for instance when constraints of nonholonomic type are present (as it happens when considering rolling in 3D between fingers and objects). Further efforts are necessary to capture the wealth of possibilities offered by the nonlinear nature of the problem.

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