H_2–PSEUDO OPTIMAL MODEL FOLLOWING: A GEOMETRIC APPROACH

G. Marro * F. Morbidi ** D. Prattichizzo **

* *Dipartimento di Elettronica, Informatica e Sistemistica, Università di Bologna, Viale Risorgimento 2, 40136 Bologna - Italy*
E-mail: gmarro@deis.unibo.it
** *Dipartimento di Ingegneria dell'Informazione, Università di Siena, Via Roma 56, 53100 Siena - Italy*
E-mail: {morbidi,prattichizzo}@dii.unisi.it

Abstract: The paper proposes a very straightforward change of the general layout of the geometric approach to the feedforward and feedback model following problems for nonminimum-phase systems. The design technique relies on the replacement of the output matrix of the controlled system with an equally dimensioned matrix ensuring H_2-optimality in the standard disturbance decoupling problem, while maintaining the relative degree and the steady-state gain of the original system. The new matrix is derived by applying the standard geometric approach tools to the Hamiltonian system instead of the original plant.

Keywords: Linear systems, Geometric approach, Model following, Nonminimum-phase systems, Optimal tracking

1. INTRODUCTION

The *model following problem* (MFP), i.e. that of synthesizing a state or output feedback control law for a given plant in order to make the impulse response matrix of the compensated system exactly equal to that of a prespecified model, has been studied for different classes of systems since the early 1970's (Wolovich, 1972; Wang and Desoer, 1972; Moore and Silverman, 1972; Wang and Davison, 1972). These early investigations were primarily concerned with problem solvability under various hypotheses.
In (Morse, 1973), the MFP was precisely stated and necessary and sufficient geometric conditions for its resolution using dynamic compensation and regular state feedback were provided. The internal stability problem was discussed, but only few preliminary results were given.
Several papers extended and refined Morse's early results. In (Anderson and Scott, 1977), the set of all stable solutions to the MFP was presented in a parametric form. In (Malabre, 1982), the extension to quadruples (A, B, C, D) was proposed and necessary and sufficient conditions under which solutions exist using regular output feedback were provided. In (Malabre and Kučera, 1984), a new geometric proof for a solvability condition of the MFP expressed in terms of the infinite zero structure was presented.
Recently in (Marro and Zattoni, 2005), a new procedure to synthesize minimal-order regulators for exact model following by output feedback with stability has been introduced. The approach, completely embedded in the geometric framework, exploits the properties of self-bounded controlled invariant subspaces and it provides an effective treatment of nonminimum-phase systems.
However the design strategy in (Marro and

Zattoni, 2005) requires that all unstable invariant zeros of the nonminimum-phase system are replicated in the model, thus significantly constraining the performance of the resulting overall feedforward (or feedback) system.

To solve this problem, a new design layout of the geometric approach to the feedforward and feedback model following problems is proposed in this paper. Particularly inspired by the recent investigations on H_2 disturbance decoupling (Saberi *et al.*, 1995; Marro *et al.*, 2002*b*), the exact model following problem is reformulated here in a H_2-minimal framework. The relative degree condition of the plant does not guarantee in general the existence of a solution of this more general problem. In these cases (namely when the relative degree is greater or equal than 2) a *pseudo-optimal* solution owning some special features is proposed. The MFP is solved for a new controlled system where the output matrix is replaced with an equally sized one. This allows us to approach the H_2 control problem as a standard disturbance decoupling while keeping the same relative degree and the steady-state gain of the original system. The new matrix is derived by applying the standard geometric approach tools to the Hamiltonian system instead of the original plant and this results in a tractable procedure easy to implement in Matlab® using the basic routines of the geometric approach (Marro, 2004).
Note that in (Stoorvogel, 1992; Stoorvogel *et al.*, 1993) geometric notation was also used to deal with singular H_2 control problems with state and measurement feedback. However the procedure therein adopted, based on LMI's and a special coordinate basis, is very far from the standard geometric approach computational environment and does not consider feedback and possible inclusion of the internal model in the controller.
The rest of the paper is organized as follows. In Sect. 2 the notation and some basic definitions of the geometric approach are introduced. In Sect. 3 the exact model following problem as presented in (Marro and Zattoni, 2005) is briefly reviewed. In Sect. 4 the H_2-pseudo optimal feedforward model following problem is stated and a geometric solution is provided. In Sect. 5 the previous results are extended to the feedback model following problem. In Sect. 6 a numerical example illustrates the theory and shows the effectiveness of the proposed approach. In Sect. 7 the major contributions of the paper are summarized and some concluding remarks are provided.

2. NOTATION

Through the paper, $\mathbb{R}^n$ stands for the set of all n-tuples of real numbers and $\mathbb{C}_g$ denotes the open left-half complex plane. Script capitals, like $\mathcal{V}$, denote vector spaces and subspaces, while italic capitals, like A, denote matrices and linear maps. $\dim\mathcal{V}$ and $\mathcal{V}^\perp$ denote the dimension and the orthogonal complement of subspace $\mathcal{V}$. $\operatorname{im} A$, $\ker A$, A^T and A^{-1} are used respectively for the image, the null space, the transpose and the inverse of matrix A.
Regarding the standard notation of the geometric approach (Wonham, 1985; Basile and Marro, 1992), $\mathcal{V}^*$ and $\mathcal{S}^*$ stand for the maximum output nulling controlled invariant and the minimum input containing conditioned invariant of an LTI system Σ, while $\mathcal{Z}(\Sigma)$ is used for the set of all the invariant zeros of Σ.

3. A REVIEW OF THE EXACT MODEL FOLLOWING PROBLEM

In this section the solution to the exact model following problem as presented in (Marro and Zattoni, 2005) is briefly recalled. The standard layout of feedforward model following is shown in Fig. 1. The *plant* Σ is the continuous-time LTI system

$$\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) \\ y(t) &= C\,x(t) \end{aligned} \tag{1}$$

where $x \in \mathcal{X} = \mathbb{R}^n$, $u \in \mathbb{R}^p$ and $y \in \mathbb{R}^p$ denote the state, the control input and the controlled output, respectively. The *model* Σ_m is the continuous-time LTI system

$$\begin{aligned} \dot{x}_m(t) &= A_m\,x_m(t) + B_m\,h(t) \\ y_m(t) &= C_m\,x_m(t) \end{aligned} \tag{2}$$

where $x_m \in \mathcal{X}_m = \mathbb{R}^q$, $h \in \mathbb{R}^p$ and $y_m \in \mathbb{R}^p$ denote the state, the exogenous input and the measurable output.

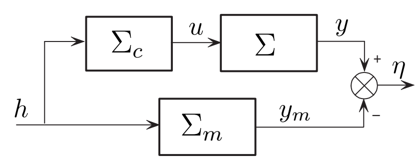

Fig. 1. Feedforward model following.

The exact feedforward model following problem is stated as follows.

Problem 1. (Exact feedforward model following) Refer to the block diagram shown in Fig. 1. Design a linear dynamic feedforward compensator $\Sigma_c \equiv (A_c, B_c, C_c, D_c)$ such that the forced evolution of η is zero for every input h.

Due to lack of space, generality will herein be sacrificed to simplicity. Both Σ and Σ_m are assumed to be minimal, stable, with no common poles and zeros, square, left and right invertible, (i.e., such that $\mathcal{V}^* \cap \mathcal{S}^* = \{0\}$, $\mathcal{V}^* + \mathcal{S}^* = \mathcal{X}$ and

$\mathcal{V}_m^* \cap \mathcal{S}_m^* = \{0\}$, $\mathcal{V}_m^* + \mathcal{S}_m^* = \mathcal{X}_m$), and steady-state completely output controllable (i.e., with the matrices $C\,A^{-1}\,B$ and $C_m\,A_m^{-1}\,B_m$ finite and nonsingular). Σ is supposed to be observable as well.
Problem 1 is solvable if

$$\mathcal{Z}(\Sigma) \subseteq \mathbb{C}_g \tag{3}$$

$$\rho(\Sigma) \leq \gamma(\Sigma_m) \tag{4}$$

i.e., if Σ is minimum-phase and has a global relative degree ρ not greater than the minimum delay γ of Σ_m. Recall that the global relative degree is computed in geometric terms as the minimum value of i such that $\mathcal{V}^* + \mathcal{S}_i = \mathcal{X}$, where $\mathcal{S}_i$ denotes the i-th subspace in the well-known sequence for computing $\mathcal{S}^*$ (Basile and Marro, 1992). The minimum delay of a triple is defined as the minimum value of i such that $C\,A^i\,B$ is nonzero. Conditions (3), (4) are presented as only sufficient, but indeed they also are almost necessary, i.e., generally also necessary, except for some pathological cases whose investigation may be interesting, but beyond the introductory scope of this paper. The requirement that Σ is stable is not mandatory in practice since it is always possible to use a stabilizing feedback through a full or reduced order observer Σ_o as shown in Fig. 2, thus replacing it with a new stable system denoted here and thereafter by Σ, Σ_o. In fact, under the minimality assumption, Σ is both reachable and observable.
The following property holds (refer to (Basile and Marro, 1992)).

Property 1. The overall system whose block diagram is shown in Fig. 2, with algebraic state feedback F through an observer, has the eigenvalues of $A + BF$ as poles and the invariant zeros consisting of the invariant zeros of Σ and the poles of Σ_o.

In fact, due to lack of controllability of the observer, the overall system is not minimal, but it is input-output equivalent to a n-th order system with arbitrary poles and with the same invariant zeros as Σ.
It is well known that the exact feedforward model following problem with stability is a particular case of the standard (exact) measurable disturbance decoupling problem with stability (MDDPS) for an extended system $\overline{\Sigma}$ also including the model, as shown in Fig. 3. The following property recalls some features of the solution.

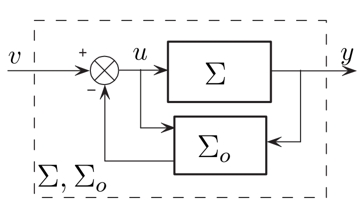

Fig. 2. Using a stabilizer.

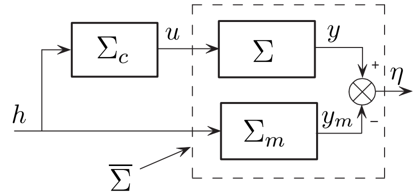

Fig. 3. Model following as a MDDPS.

Property 2. The poles of the compensator Σ_c shown in Fig. 1, that is obtained with the standard geometric procedure to solve the MDDPS, consist of the poles of Σ_m and the invariant zeros of Σ, while the invariant zeros of Σ_c coincide with the poles of Σ (or the poles assigned by state feedback through an observer as shown in Fig. 2).

If Σ is not minimum-phase, a possible design policy considered in (Marro and Zattoni, 2005), consists of placing a suitable replica of all its unstable invariant zeros in the model. The main drawback of this solution is that it introduces a significant constraint on the control system performance. To overcome this drawback, a possible alternative strategy is presented in the next section as the main result of the present paper.

4. H_2-PSEUDO OPTIMAL FEEDFORWARD MODEL FOLLOWING

When the plant Σ is nonminimum-phase, a possible strategy consists in reformulating Problem 1 in a H_2-minimal context. Unfortunately condition (4) does not guarantee in general the existence of a solution of this more general problem. Actually, it can be proved that when $\rho(\Sigma) \geq 2$ the problem does not admit solutions of the type sketched in Fig. 3 (since Σ_c cannot generate distributions). Hence, a suitable *pseudo-optimal* solution to the problem is herein proposed.

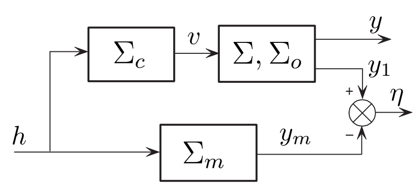

Fig. 4. Modified feedforward model following.

Refer to the modified feedforward scheme shown in Fig. 4. Replace matrix C with another matrix C_1 such that the new system Σ_1, corresponding to the triple (A, B, C_1), is minimum-phase, the ℓ_2 norm of output y_1 is minimal for the maximum set of initial states compatible with a control without distributions and the steady-state gain from v to y_1 is equal to that from v to y. This auxiliary output may be provided by a full or reduced order observer Σ_o connected to the input and output of the original plant (see Fig. 2). Σ_o could also be used to stabilize the system or give it a more favourable pole placement for optimal (H_2 or H_∞) rejection of unaccessible disturbances directly acting on Σ. C_1 can be computed as shown in the constructive proof of the following theorem.

Theorem 1. Refer to system Σ and assume that it is nonminimum-phase, with no purely imaginary invariant zeros. A unique matrix C_1 exists such that:
1 - the exact disturbance decoupling problem with state feedback for any input disturbance matrix H such that $\mathrm{im} H \subseteq \mathcal{V}_1^*$, where $\mathcal{V}_1^*$ is referred to the triple (A, B, C_1), corresponds to a minimal H_2-norm solution for the triple (A, B, C);
2 - the number of outputs of (A, B, C_1) is equal to those of (A, B, C);
3 - the triple (A, B, C_1) is minimum-phase;
4 - the triple (A, B, C_1) has the same global relative degree as (A, B, C);
5 - the steady-state gain of (A, B, C_1) is equal to that of (A, B, C).

Proof: Following a by now standard approach, let us recall that any minimum H_2-norm problem is equivalent to a standard disturbance decoupling problem for the Hamiltonian system $\widehat{\Sigma}$ defined by

$$\begin{aligned} \dot{\hat{x}}(t) &= \widehat{A}\,\hat{x}(t) + \widehat{B}\,u(t) \\ 0 &= \widehat{C}\,\hat{x}(t) \end{aligned} \qquad \hat{x} = \begin{bmatrix} x \\ p \end{bmatrix}$$

with

$$\widehat{A} = \begin{bmatrix} A & 0 \\ -C^{\mathrm{T}} C & -A^{\mathrm{T}} \end{bmatrix}, \quad \widehat{B} = \begin{bmatrix} B \\ O \end{bmatrix} \qquad (5)$$
$$\widehat{C} = \begin{bmatrix} O & B^{\mathrm{T}} \end{bmatrix}.$$

It is well known ((Marro *et al.*, 2002*a*), adapted from discrete to continuous-time systems) that if Σ is left invertible, $\widehat{\Sigma}$ is both right and left invertible, and if Σ is left and right invertible, $\mathcal{Z}(\widehat{\Sigma})$ consists of all the elements of $\mathcal{Z}(\Sigma)$ and their opposite. Denote by $\widehat{\mathcal{V}}^*$ the maximum output nulling controlled invariant subspace of $\widehat{\Sigma}$, whose internal eigenvalues are the elements of $\mathcal{Z}(\widehat{\Sigma})$, and by

$$\widehat{\mathcal{V}}_g^* = \mathrm{im} \begin{bmatrix} V_1 \\ P_1 \end{bmatrix}$$

its restriction to the stable internal eigenvalues. Clearly, the dimension of $\widehat{\mathcal{V}}_g^*$ is equal to that of $\mathcal{V}^*$. $\mathcal{V}_1^* := \mathrm{im} V_1$ is an (A, B)-controlled invariant that, by construction, satisfies property 1 of the statement. The dimension of $\mathcal{V}_1^*$ is equal to that of $\mathcal{V}^*$, but $\mathcal{V}_1^*$ is not contained in $\ker C$ if and only if Σ is nonminimum-phase.
Let us refer the symbol $\mathcal{S}^*$ to the triple (A, B, C) and $\mathcal{S}_1^*$ to (A, B, C_1). It will be shown now that

$$\mathcal{V}_1^* \cap \mathcal{S}^* = \{0\}. \qquad (6)$$

From (5) and from the inclusion $\widehat{\mathcal{V}}_g^* \subseteq \widehat{\mathcal{V}}^*$, it follows that

$$\mathrm{im}\, B \subseteq (\mathrm{im}\, P_1)^\perp. \qquad (7)$$

Moreover, from the cost formulation of the optimal control problem, it is well known that $V_1^T P_1 \neq 0$ since in our case the cost (the H_2 norm) is nonzero, and consequently

$$\mathrm{im}\, V_1 \nsubseteq (\mathrm{im}\, P_1)^\perp. \qquad (8)$$

From (7) and (8), one obtains $\mathrm{im}\, V_1 \cap \mathrm{im}\, B = \{0\}$, hence

$$\mathcal{V}_1^* \cap \mathcal{S}_1^* = \{0\}. \qquad (9)$$

To prove (6), it still remains to show that $\mathcal{S}_1^* = \mathcal{S}^*$. From the algorithm for the computation of the minimum input containing conditioned invariant (Basile and Marro, 1992), it follows that

$$\mathcal{S}_1^* \supseteq \mathcal{S}^* \qquad (10)$$

while from a simple consideration on subspaces dimensions, i.e. $\dim \mathcal{S}_1^* = \dim \mathcal{S}^*$, from (10) it follows that $\mathcal{S}_1^* = \mathcal{S}^*$. This completes the proof of (6).
Let $\mathcal{C} := \ker C$. To conclude the proof of the theorem, consider the identity

$$\mathcal{C} = \mathcal{V}^* + (\mathcal{S}^* \cap \mathcal{C})$$

obtained by intersection of $\mathcal{V}^* + \mathcal{S}^* = \mathcal{X}$ with $\mathcal{C}$, and set

$$\mathcal{C}_1 = \mathcal{V}_1^* + (\mathcal{S}^* \cap \mathcal{C}). \qquad (11)$$

Owing to (6) and being the dimensions of $\mathcal{V}^*$ and $\mathcal{V}_1^*$ equal, the subspaces $\mathcal{C}_1$ and $\mathcal{C}$ have the same dimension. Let $\bar{C}_1$ be the transpose of any basis matrix of the orthogonal complement of the subspace $\mathcal{C}_1$ defined in (11), hence having $\mathcal{C}_1$ as its kernel. The new matrix C_1 is defined as

$$C_1 = G\,\bar{G}^{-1}\,\bar{C}_1 \qquad (12)$$

where G denotes the steady-state gain of the triple (A, B, C) and $\bar{G}$ that of the triple $(A, B, \bar{C}_1)$.
Property 1 in the statement is satisfied because the maximum (A, B)-controlled invariant contained in $\ker C_1 = \mathcal{C}_1$ is $\mathcal{V}_1^*$, since $\mathcal{V}_1^* \cap \mathcal{S}^* = \{0\}$. Property 2 is satisfied since $\mathcal{C}$ and $\mathcal{C}_1$ have the same dimension. Property 3 is satisfied since the invariant zeros of (A, B, C_1) are the internal eigenvalues of $\mathcal{V}_1^*$, stable by construction. Property 4 is satisfied since both the triples (A, B, C_1) and (A, B, C) have $\mathcal{S}^*$ as the minimum conditioned invariant containing $\mathrm{im} B$ and, since both $\mathcal{V}_1^*$ and $\mathcal{V}^*$ have null intersection with $\mathcal{S}^*$, the dimensions of $\mathcal{V}_1^* + \mathcal{S}_i$ and $\mathcal{V}^* + \mathcal{S}_i$ are each other equal for all i. Property 5 is due to relation (12). Uniqueness of matrix C_1 is due to uniqueness of all the subspaces involved in the constructive procedure described above and to relation (12) that sets a unique value for the steady-state gain of (A, B, C_1). ■

Remark 1. It can be proven that the original triple (A, B, C) and the corresponding minimum-phase triple (A, B, C_1) have equal H_2 norms. However the proof of this property is beyond the scope and space of this paper.

Remark 2. It can be easily shown that it is possible to define a computation procedure for a different matrix C_1 preserving the vector relative degree of Σ (not the global relative degree as above), and also producing H_2-pseudo optimal model following, but missing the steady-state zero tracking error requirement.

5. H_2-PSEUDO OPTIMAL FEEDBACK MODEL FOLLOWING

The main contribution to exact model following is a very straightforward solution for the feedback connection shown in Fig. 5. In fact, it has also been pointed out in (Marro and Zattoni, 2005) that exact feedback model following can be obtained with some simple manipulation from the feedforward solution. Moreover a multiple internal model with poles at the origin can be imposed in the feedback regulator Σ_c if Σ_m is suitably chosen with this aim.

Similarly to Fig. 4, consider the modified scheme shown in Fig. 6, where the new output y_1 is provided by the observer Σ_o. Note that replacing the feedback connection in Fig. 6 with that shown in Fig. 7 does not affect the structural properties of the system since signals y and y_1 are identically equal, but may affect stability. The new block diagram represents a feedforward model following problem. In fact, note that h is obtained as the difference of r (applied to the input of the model) and y_m (the output of the model). This corresponds to the parallel connection of Σ_m and the opposite of the identity matrix, that is invertible, having zero relative degree. Its inverse is Σ_m with a feedback connection through the identity matrix, as shown in Fig. 8.

It is thus evident that from a structural point of view, the modified feedback model following in Fig. 6 is equivalent to a modified feedforward problem which refers to the model $\Sigma'_m \equiv (A_m + B_m C_m,\, B_m,\, C_m)$ (see Fig. 8).

It is then possible to bring back the feedback model following problem to the feedforward problem discussed in Sect. 4 by means of the above algebraic transformations, that guarantee existence of equal solutions of the differential equations describing the overall two systems (called herein *structural equivalence* of the two involved systems), but not guarantee stability. In fact, the

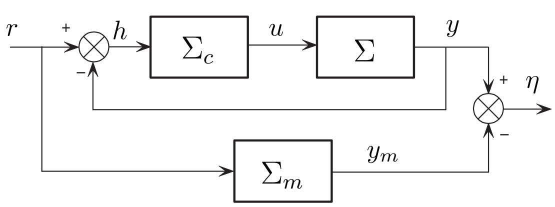

Fig. 5. Exact feedback model following.

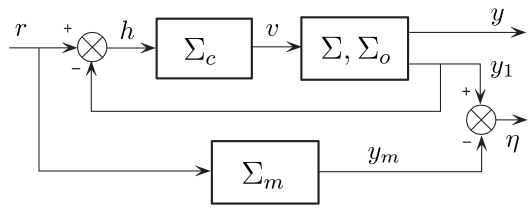

Fig. 6. Modified feedback model following.

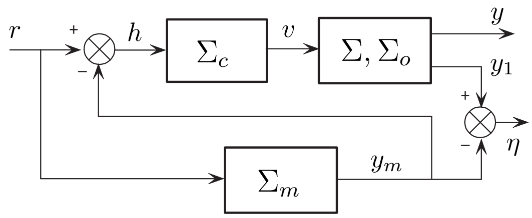

Fig. 7. A structurally equivalent connection.

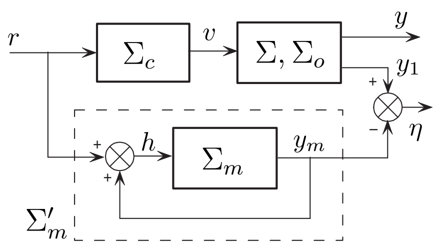

Fig. 8. Block diagram for the equivalent feedforward model following.

condition $\eta(t)=0$ for all $t \geq 0$ in the modified system may be obtained as the difference of diverging signals y_1 and y_m. However, stability is recovered when going back to the original feedback connection represented in Fig. 5, that ensures that all the modes present in the impulse response of Σ_m are stable.

6. AN EXAMPLE

Suppose that the plant Σ is defined by

$$A = \begin{bmatrix} -8.95 & -6.45 & 0 & 0 \\ 2.15 & -0.35 & 0 & 0 \\ -10.89 & -40.94 & -16.10 & -7.95 \\ 8.17 & 28.87 & 7.07 & -0.20 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 10 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 5 & 6 \end{bmatrix}$$

and the model Σ_m is

$$A_m = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}, \; B_m = C_m = \begin{bmatrix} 1.4142 & 0 \\ 0 & 1.7321 \end{bmatrix}.$$

The plant Σ is nonminimum-phase being $\mathcal{Z}(\Sigma) = \{14.7907, -9.7907\}$ and the model is SISO-parallel, hence steady-state decoupling.

If the constructive procedure detailed in the proof of Theorem 1 is applied to Σ (for the sake of simplicity, no observers Σ_o are implemented), we obtain

$$C_1 = \begin{bmatrix} 1.5452 & 2.6986 & 3.3116 & 4.1613 \\ 1.6237 & 0.5178 & 4.7850 & 5.8887 \end{bmatrix}$$

with $\mathcal{Z}(\Sigma_1) = \{-14.7907, -9.7907\}$. Fig. 9 shows the step response of the system

$$\widetilde{A} = \begin{bmatrix} A & BC_c & O \\ O & A_c & O \\ O & O & A_m \end{bmatrix}, \quad \widetilde{B} = \begin{bmatrix} BD_c \\ B_c \\ B_m \end{bmatrix}$$

$$\widetilde{C} = \begin{bmatrix} C & O & -C_m \end{bmatrix}, \qquad \widetilde{D} = O$$

whose output is the actual tracking error. From an inspection of the four plots, it is evident that the proposed H_2-pseudo optimal solution guarantees an excellent transient response.

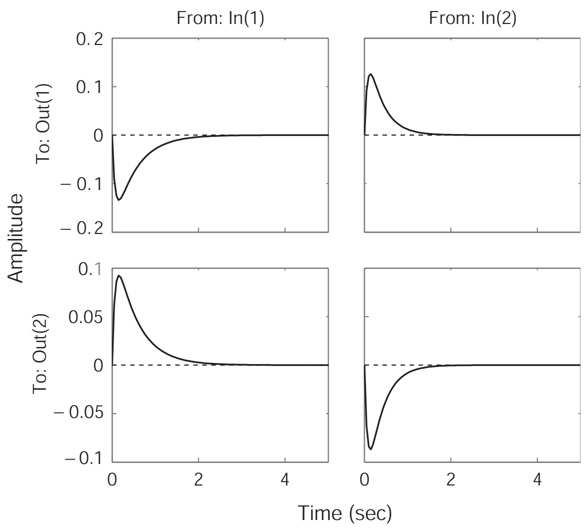

Fig. 9. The actual step response tracking error.

7. CONCLUSIONS

In this paper a new geometric solution to the feedforward and feedback model following problems is proposed. The design strategy is based on the replacement of the output matrix of the controlled system with an equally dimensioned matrix ensuring H_2-optimality in the standard disturbance decoupling problem, while maintaining the global relative degree and the steady-state gain of the original system. The new matrix is derived by applying the standard geometric approach tools to the Hamiltonian system instead of the original plant and using some refinements to adapt the solution of the H_2-optimal decoupling problem to the requirements of model following.

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