On Motion and Force Controllability of Precision Grasps with Hands Actuated by Soft Synergies

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***Abstract*—To adapt to many different objects and tasks, hands are very complex systems with many degrees of freedom (DoFs), sensors, and actuators. In robotics, such complexity comes at the cost of size and weight of the hardware of devices, but it strongly affects also the ease of their programming. A possible approach to simplification consists in coupling some of the DOFs, thus affording a reduction of the number of effective inputs, and eventually leading to more efficient, simpler, and reliable designs. Such coupling can be at the software level, to achieve faster, more intuitive programmability or at the hardware level, through either rigid or compliant physical couplings between joints. Physical coupling between actuators and simplification of control through the reduction of independent inputs is also an often-reported interpretation of human hand movement data, where studies have demonstrated that few "postural synergies" explain most of the variance in hand configurations used to grasp different objects. Together with beneficial simplifications, the reduction of the number of independent inputs to a few coupled motions or "synergies" has also an impact on the ability of the hand to dexterously control grasp forces and in-hand manipulation. This paper aims to develop tools that establish how many synergies should be involved in a grasp to guarantee stability and efficiency, depending on the task and on the hand embodiment. Through the analysis of a quasi-static model, grasp structural properties related to contact force and object motion controllability are defined. Different compliant sources are considered, for a generalization of the discussion. In particular, a compliant model for synergies assumed, referred to as "soft synergies," is discussed. The controllable internal forces and motions of the grasped object are related to the actuated inputs. This paper investigates to what extent a hand with many joints can exploit postural synergies to control force and motion of the grasped object.**

Manuscript received February 13, 2013; accepted July 14, 2013. Date of publication August 15, 2013; date of current version December 2, 2013. This paper was recommended for publication by Associate Editor T. Simeon and Editor W. K. Chung upon evaluation of the reviewers' comments. This work was supported in part by the European Commission Collaborative Project 248587 and "The Hand Embodied," within the FP7-ICT-2009-4-2-1 Program "Cognitive Systems and Robotics."

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Digital Object Identifier 10.1109/TRO.2013.2273849

***Index Terms*—Compliance, controllability, grasping, robotic hands.**

I. Introduction

ROBOTIC hands have many degrees of freedom (DoFs) distributed among several kinematic chains, the fingers. The complexity of the mechanical design is needed to adapt hands to the many kinds of tasks required in unstructured environments. Over the years, roboticists have attempted to imitate the human hand in terms of dexterity and adaption capabilities. Some remarkable examples of robotic hand design are the UTAH/MIT hand [34], the DLR hand II [14], and the JPL/Stanford hand [37]. One of the main issues in designing and controlling robotic hands is that a large number of motors is needed to fully actuate the DoF but this comes at the cost of size, complexity, and weight of the device. This disadvantage could be overtaken if robotic hands were actuated and controlled by a reduced number of inputs, thus resulting in their being more efficient, simpler, and reliable than their fully actuated alternatives, as shown in [3], [8], and [12].

A promising direction in the design of robotic hands focuses on two key principles: underactuation and passive mechanical adaptation. Underactuation in robotics [11] essentially refers to systems that have more DoF than actuators. More specifically, in this type of system, the DoFs are more than the degrees of actuation [10]. In grasping and manipulation tasks, the unactuated joints often have elastic elements [11], [25], [45]; however, there are also underactuated hands without elastic elements, e.g., the solution proposed in [24]. If elastic elements are present in nonactuated hand joints, then we should consider these joints as uncontrollable or passively driven instead of unactuated [6], [9], [40]. The presence of passively actuated joints allows the hand to self-adapt to the surface in a simple and robust way [8], [35]. A quasi-static model of underactuated compliant robotic hands is described in [20]. Grasp properties with underactuated hands, and in particular, grasp stiffness are analyzed in [40]. An anthropomorphic underactuated robotic hand with 15 DoFs and a single actuator is described in [33].

A simplified control seems to inspire also biological systems, and in particular, motor control of human hands, which share with robotic ones the large number of DoFs. Studies in neuroscience [54], [55] demonstrated that a limited set of input variables, called postural synergies, are able to describe most of the variance in hand movements and configurations in grasping tasks. Recently, these studies on human hands inspired new research on design and control strategies for robotic hands whose main issue is to achieve a tradeoff between simplicity, gained through synergy-based control, and versatility [13], [21]. One example of a robotic hand whose design is based on the adaptive synergy concept has been recently introduced in [16] and developed in [15]. In [21] and later in [22], the synergy concept has been applied to control different hand models: a simple gripper, the Barrett hand, the DLR hand, the Robonaut hand, and the human hand model. In [13] a robotic hand design able to match postural synergies mechanically coupling motion of the single joints is proposed. Postural synergies in robotic hands allow control of the whole device, with n_q joints, through a lower dimension set of actions $n_z \leq n_q$: indicating with $\dot{q}$ hand joint velocities, the following relationship can be defined:

$$\dot{q} = S\dot{z} \tag{1}$$

where S is the synergy matrix, and $\dot{z}$ represents synergy velocities. Columns of the matrix of synergies $S \in \Re^{n_q \times n_z}$ represent the so-called postural synergies, also referred to as eigengrasps in the literature, e.g., in [21], in other terms, the joint velocities that are obtained acting on each single synergy $\dot{z}_i$.

In human hands, the synergies can be evaluated analyzing measures of hand postures, for instance, performing a principal component analysis (PCA) of hand postures during grasping operations in which hand configuration is experimentally measured, as described in [54]. In robotic hands, where a mechanical coupling between joints postural synergies is present, synergies can be derived from the kinematic analysis of joint couplings and constraints. In the artificial, hand synergies can be introduced also at the control level. In this case, the coupling between hand joints is managed by the hand control system. In [56], an impedance control for multifingered robotic hands based on the definition of postural synergies was proposed. It was implemented on the DLR hand II [14], whose postural synergies were defined performing a PCA analysis on a wide database of grasp configurations. In [31], a procedure (based on the task-object space) to map human hand synergies on robotic hands, even with a kinematic structure very dissimilar from the anthropomorphic one, is proposed and discussed. In [27], the postural synergies configuration subspace of the University of Bologna hand (UBH) [36] are evaluated. This study is based on the kinematic structure of the robotic hand and on the taxonomy of the grasp of common objects. In [30], the dynamic optimization of tendon tensions in anthropomorphically designed hands with rolling constraints is analyzed [7].

Intuitively, reducing the number of control inputs, from n_q actuated joints to n_z synergies, may reduce the dimension of the force and motion controllability subspaces, thus compromising the dexterity of the given robotic grasp. However, this is not true in general and strongly depends on the column space of synergy matrix S. Some of the main questions to answer when interpreting the motion and force control in the light of synergies are as follows. How many synergies have to be involved in a given grasp? Which contact forces are controllable when acting on synergies instead of each single actuator independently? Is a synergy based actuation of the robotic hand sufficient to guarantee a stable and efficient grasp? What kind of force feedback information is needed to implement the feedback controller based on synergy? In this paper, we tried to find some answers to these questions, extending to the synergy controlled hands results previously described in [1], [2], and [49].

If we defined the actual joint variables, according to (1) as a linear combination of synergies, as, e.g., in [13] and [21], hand postures would rigidly lie on the manifold defined by the synergy matrix, and consequently, the hand would rigidly move along given directions in the joint space thus making it impossible to grasp objects with different shapes and dimensions. This does not happen to the human hand, which is able to easily grasp different objects. To gain the same level of adaptability and robustness in robotics, we need to use a definition of synergies based on compliance whereby (1) is treated as a singular case. In this paper, synergies are defined according to a compliant model of joint torques. Synergies represent a joint displacement aggregation corresponding to a reduced dimension representation of the hand *reference* movements and the actual hand posture will differ from the reference one because of its compliance. We defined this type of hand joint coupling as *compliant postural synergies*, or *soft synergies*, the concept of soft synergies was first introduced in [28] and [51], then developed in [4], [29], [50], and [52].

Compliance is one of the most important aspects to consider for characterizing the grasp of a robotic hand on an object or on a tool, especially when fine manipulation or high precision is required, e.g., in assembling components. In the analysis of grasping, in general, different compliance sources have to be taken into account: contact stiffness, due, for example, to fingertip elasticity; actuator stiffness, given by the position control static gain; and structural compliance, due to the mechanical deformation of hand elements (joints, drive cables, links, etc.) [23].

A preliminary version of the study on hands with postural synergies was presented in [51]. With respect to that paper, the present one adds details to the grasp model, including terms that were not considered in [51], e.g., geometrical stiffness terms coming from hand Jacobian derivatives [18], [23], [47], and provides a complete solution to the quasi-static model, defining the mapping matrices between the input reference systems and grasp configuration and forces. The evaluation of rigid-body motion subspace has been simplified with respect to [51], using results coming from the quasi-static solution. Furthermore, more examples are provided, including an anthropomorphic hand model.

In cases where phalanges have a contact surface, during grasp and manipulation operations, it is very common that some of the contacts are rolling [41], [46]. However, since the attention in this paper is focused on the definition of structural grasp properties related to the presence of a limited number of actuators, for the sake of simplicity, the study is limited to precision grasps with point contact and friction [53], and the rolling in contact is not considered. Rolling contact modeling is important in the dynamic evolution simulation of grasp [5], [30]; however, in a quasi-static contest, it has effect only in the definition of geometrical stiffness terms [17].

The paper is organized as follows. Section II introduces the main definitions and equations necessary in grasp analysis. Section III describes the contact forces and object motions controllable by the input synergies. Section IV discusses the properties of force and rigid-body motion subspaces. Section V

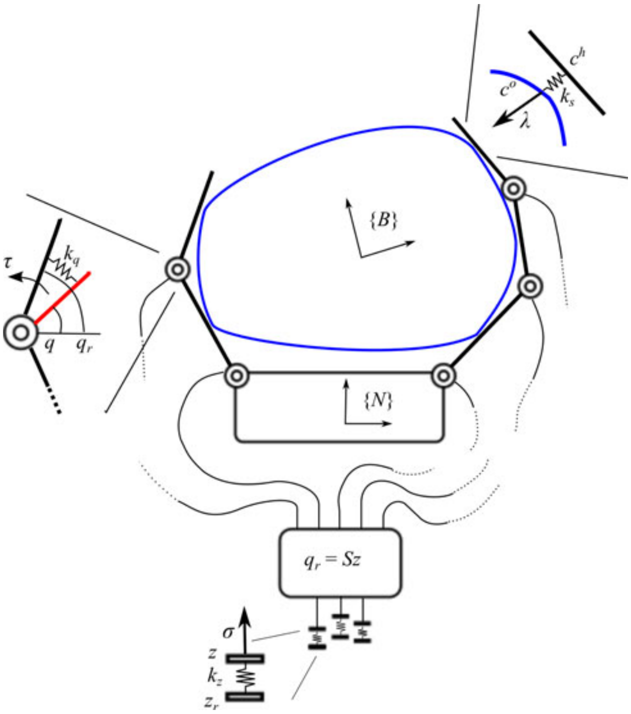


Fig. 1. Hand–object grasp with postural synergies: Main quantities.

shows the results described in the preceding sections with some numerical examples, relative to hands with increasing complexity: a simple gripper, a robotic hand with kinematics similar to the Barrett hand, and an anthropomorphic hand. Finally, Section VI concludes the paper. In the Appendix, the details of some mathematical steps are provided.

II. Modeling Hands with Soft Synergies

Consider a robotic hand grasping an object as in Fig. 1. Let $\{N\}$ represent the inertial frame fixed in the workspace and let frame $\{B\}$ be fixed to the object. Let $u \in \Re^{n_d}$ denote the vector that describes the position and orientation of $\{B\}$ relative to $\{N\}$, $n_d = 3$ for planar systems, $n_d = 6$ for spatial systems, $\xi \in \Re^{n_d}$ the object twist, and $w \in \Re^{n_d}$ the wrench applied to the object, all expressed with respect to $\{B\}$.

Let n_c be the number of contact points, $\{C_i^h\}$ the reference frame on the ith contact point, connected to the hand, and $\{C_i^o\}$ the corresponding reference frame connected to the object. Let $\hat{c}_i^h, \hat{c}_i^o \in \Re^{n_d}$ denote the vector that describes the position and orientation of $\{C_i^h\}$ and $\{C_i^o\}$, respectively, relative to $\{B\}$.

Let $w_{c_i^h}^{c_i^o} \in \Re^{n_d}$ be the wrench that the hand exerts on the object on the ith contact point, whose components are expressed with respect to $\{C_i^o\}$.

A suitable contact model is introduced to define contact constraints and forces [53]: for each contact i, the contact force vector $\lambda_i \in FC_i \subset \Re^{l_i}$ is defined, in which l_i depends on the contact type and FC_i represents the subspace of allowable contact forces. For example, for a point contact with friction (PCWF) model, $l_i = 3$ and FC_i is the so-called friction cone, defined as

$$FC_i = \ \lambda_i \in \Re^{l_i} : \sqrt{\lambda_{i,1}^2 + \lambda_{i,2}^2} \leq \mu\lambda_{i,3}$$

where $\lambda_{i,1}$ and $\lambda_{i,2}$ are the contact force components orthogonal to the contact point normal direction, $\lambda_{i,3}$ represents the normal contact force, and μ is the friction coefficient [44].

The object static equilibrium equation can be expressed with respect to the object reference frame $\{B\}$ as

$$w + G\lambda = 0 \tag{2}$$

in which $\lambda = [\lambda_1^{\mathrm{T}}, \ldots, \lambda_{n_c}^{\mathrm{T}}]^{\mathrm{T}}$, $\lambda \in \Re^{n_l}$, where $n_l = \sum_{i=1}^{n_c} l_i$, and $G \in \Re^{n_d \times n_l}$ is the Grasp matrix [44].

The general solution of (2), assuming that w is in the column space of G, $\mathcal{R}(G)$, is $\lambda = -G^{+}w + A\chi$, where G^{+} is a generic right inverse of the grasp matrix and $A \in \mathbb{R}^{n_l \times n_h}$ is a matrix whose columns form a basis of the nullspace of G, $\mathcal{N}(G)$, and the vector $\chi \in \mathbb{R}^{n_h}$ parameterizes the homogeneous part of the solution. The term $A\chi$ represents the solution to (2) when no external load w is applied, and is usually referred to as *internal forces*. For general grasp kinematics, and in particular in hands with few actuators, controlling internal forces is not straightforward since the number of internal forces directions, i.e., the dimension of the subspace $\mathcal{N}(G)$, turns out to be larger than the number of controlled joint actions [1], [2].

According to the defined contact model, we can highlight the constrained components of the relative motion in two vectors $v_{c^o}^{c^o}, v_{c^h}^{c^o} \in \Re^{n_l}$. A linear relationship exists between the contact twist components constrained by the contact model and the object twist, i.e., $v_{c^o}^{c^o} = G^{\mathrm{T}}\xi$. Similarly, we can select from the vectors $\hat{c}_i^o$ and $\hat{c}_i^h$ the components c_i^o and c_i^h, constrained by the contact model and collect them in the vectors $c^o = [c_1^o, \cdots, c_{n_c}^o]$ and $c^h = [c_1^h, \ldots, c_{n_c}^h]$. We can furthermore approximate the contact frame variation as a function of object configuration variation as

$$\Delta c^o = G^{\mathrm{T}}\Delta u. \tag{3}$$

Let $q = [q_1 \ \cdots \ q_{n_q}]^{\mathrm{T}} \in \Re^{n_q}$ define the vector of joint displacements. The components of contact point twists on the hand, constrained by the contact model, and expressed with respect to $\{C_i^o\}$ reference frames, can be evaluated as a function of hand joint velocities as $v_{c^h}^{c^o} = J\dot{q}$, in which $J \in \Re^{n_l \times n_q}$ represents the hand Jacobian matrix. The contact frame displacement can be expressed as a function of joint variation as

$$\Delta c^h = J\Delta q. \tag{4}$$

Considering a generic equilibrium configuration of the hand, the contact forces are balanced by the joint action $\tau \in \Re^{n_q}$, i.e.,

$$\tau = J^{\mathrm{T}}\lambda. \tag{5}$$

More details on G and J matrices evaluation of can be found in [44] and are summarized in the Appendix for the reader's convenience.

The hand forward kinematic analysis allows us to find a relationship between the synergy value z and the reference joint value q_r, that can be different from the actual one q, since a compliant actuation model is adopted for the hand joints. In the most general case, we can suppose the forward kinematic relationship to be nonlinear

$$q_r = f_z(z) \tag{6}$$

where $f_z : \Re^{n_z} \rightarrow \Re^{n_q}$ represents the kinematic map. More in general, the hand can be represented as a mechanism with n_z DoFs, in which the variables $z \in \Re^{n_z}$ represent the Lagrangian coordinates adopted as a minimum representation of hand configuration, while q_r represents a redundant configuration representation, adopted in this case to simplify the management of the contact with the grasped object.

The vector q_r represents the hand *reference* configuration, that can be different from the actual one q when contact forces are applied to the grasp. As introduced in the preceding section, if we defined the *actual* joint variables as functions of synergies, for instance according to the linear relationship in (1) as, e.g., in [13], [21], the hand would rigidly move along given directions in the joint space thus making it impossible to grasp objects with different shapes and dimensions. To generalize the discussion and improve the adaptability and robustness, we need to use a definition of synergies based on compliance.

Standard analysis tools for mechanism kinematics can be adopted to define the function f_z that represents the direct kinematic relationship between redundant configuration representation and Lagrangian coordinates z [26], [57]. More practically, the function f_z can be determined by the analysis of the mechanical structure of the hand, or evaluated by means of data analysis procedures. For instance, in [54], the synergies for the human hand were evaluated performing the PCA on a set of experimental measures of hand postures, the same technique was adopted in [56] to evaluate the synergies for the DLR-II robotic hand. The kinematic relationship defined in (6) can be differentiated in order to express the joint displacement variation with respect to an initial reference condition, as a function of the synergy variation

$$\Delta q_r = S \Delta z \tag{7}$$

where $S = \frac{\partial f_z}{\partial z} \in \Re^{n_q \times n_z}$ is the *synergy matrix*, that in the more general case depends on hand configuration, and then, on z. Finally, if we consider the hand synergy actuation, in an equilibrium configuration the following relationship between joint torques τ and synergy generalized forces σ holds:

$$\sigma = S^{\mathrm{T}} \tau. \tag{8}$$

Often in robotic hands, and particularly in underactuated ones, compliance could be significant, furthermore, the introduction of compliance allows us to solve the indeterminacy of the static problem [1]. According to the definition of grasp matrix and hand Jacobian matrix previously introduced in (3) and (4), a variation of contact force can be expressed as

$$\Delta\lambda = K_s(\Delta c^h - \Delta c^o) = K_s(J\Delta q - G^{\mathrm{T}} \Delta u) \tag{9}$$

where $K_s \in \Re^{n_l \times n_l}$ is the contact compliance matrix, symmetric and positive definite.

As outlined in [23], often the structural stiffness of the links and the controllable servo compliance of the joints have the same order of magnitude of the contact stiffness. A variation of joint torques with respect to a reference initial condition can then be expressed as

$$\Delta\tau = K_q(\Delta q_r - \Delta q) \tag{10}$$

where $K_q \in \Re^{n_q \times n_q}$ is the joint stiffness matrix, symmetric and positive definite.

According to the compliant model introduced in the preceding section, referred to as *soft synergies*, a variation of the synergy actuation forces can be evaluated as

$$\Delta\sigma = K_z(\Delta z_r - \Delta z) \tag{11}$$

where $K_z \in \Re^{n_z \times n_z}$ is a symmetric and positive definite matrix that defines the synergy stiffness, and Δz_r represents the a variation of the synergy reference value.

In the following, we furthermore indicate with C_s, C_q, and C_z the compliance matrices, corresponding to stiffness matrices K_s, K_q, and K_z, respectively, i.e., $C_s = K_s^{-1}$, and so on.

III. Forces and Object Displacements Controlled by Soft Synergies

A. Quasi-Static Linearized System Equations

Let us consider an *equilibrium* configuration where an object with an external wrench w_0 is grasped by a hand whose synergy reference values are $z_{r,0}$ and the corresponding joint displacements are q_0. The contact forces in this reference equilibrium are λ_0. Starting from this equilibrium configuration, we consider a *small* variation of the input synergy reference values Δz_r, which leads to an actual variation of the postural synergies Δz, to a variation of the joint displacement Δq and a variation of contact forces $\Delta\lambda$ for the new equilibrium configuration of the quasi-static model.

Since this paper studies the effect on the grasp due to changes of the postural synergies, which play the role of controlled variables, no other actions are considered on the grasp, and we suppose that the object wrench w_0 is kept constant. If the system is asymptotically stable, after the superimposition of the input variation, it will tend to a new equilibrium configuration [44], [49].

If the new equilibrium configuration is sufficiently *near* to the reference one, we can assume that the system can be locally linearized.

Let us then consider the equilibrium equations described in the preceding section in the new equilibrium configuration. In (2), the grasp matrix is constant if rolling motion is not considered, and if the object equilibrium equation is expressed with respect to $\{B\}$ reference frame. The object equilibrium in the new configuration, with the same external wrench, can be described by the following equation:

$$0 = -G\Delta\lambda. \tag{12}$$

We observe that $\Delta\lambda \in \mathcal{N}(G)$, that is the variation of contact force due to a variation of reference synergy input belongs to the internal force subspace. As discussed in the Introduction, for the sake of simplicity, this study is limited to precision grasp with point contact and friction [53], and the rolling in contact is not considered. If it was introduced in the model, then the variation of contact points during surface rolling should be taken into account by considering also G matrix variation in the linearization of (2), as presented, e.g., in [30].

Let us then consider the hand equilibrium equation, according to (5). It is worth to noting that the hand Jacobian matrix depends on both the hand joint configuration q and on object displacement vector u, i.e., $J = J(q, u)$. The joint torque variation $\Delta\tau$ can be then expressed as

$$\Delta\tau = J^{\mathrm{T}}\Delta\lambda + K_{J,q}\Delta q + K_{J,u}\Delta u \tag{13}$$

where $K_{J,q} \in \Re^{n_q \times n_q}$ and $K_{J,u} \in \Re^{n_q \times n_d}$ represent the derivatives of hand Jacobian matrix with respect to q and u, respectively, evaluated in the reference equilibrium configuration

$$K_{J,q} = \frac{\partial(J^{\mathrm{T}}\lambda_0)}{\partial q}, \qquad K_{J,u} = \frac{\partial(J^{\mathrm{T}}\lambda_0)}{\partial u}. \tag{14}$$

It is worth observing that, even if $K_{J,q}$ and $K_{J,u}$ elements are dimensionally a stiffness, these terms do not represent physical stiffness elements, but rather they take account for the dependence of the hand Jacobian on the grasp configuration. For this reason, they are usually referred to as *geometric terms*. Furthermore, even if $K_{J,q}$ is square, it is in general nonsymmetric [19]. Both matrices depend on the initial contact force λ_0. In [17], it was proved that these terms are necessary to obtain a conservative congruence transformation from joint torques to workspace wrench.

Finally, concerning the relationship between synergy actions and joint torques, from (8), and recalling that in the more general case the synergy matrix is not constant and depends on the synergy value, we can express the variation $\Delta\sigma$ as follows:

$$\Delta\sigma = S^{\mathrm{T}}\Delta\tau + K_{S,z}\Delta z \tag{15}$$

where

$$K_{S,z} = \frac{\partial S\tau_0}{\partial z}. \tag{16}$$

Matrix $K_{S,z} \in \Re^{n_z \times n_z}$ elements are dimensionally a ratio between a generalized force and a generalized displacement, and depends on the initial torque value τ_0.

We can summarize equilibrium conditions described by (12), (13), and (15), the constitutive conditions described by (9), (10), and (11), and the congruence condition described by (7), in the following linear system:

$$A\Delta x = \Delta y \tag{17}$$

where A matrix is square and is defined as follows:

$$A = \begin{bmatrix} 0 & 0 & G & 0 & 0 & 0 \\ K_{J,u} & K_{J,q} & J^{\mathrm{T}} & -I & 0 & 0 \\ 0 & 0 & 0 & S^{\mathrm{T}} & -I & K_{S,z} \\ K_s G^{\mathrm{T}} & K_s J & I & 0 & 0 & 0 \\ 0 & K_q & 0 & I & 0 & -K_q S \\ 0 & 0 & 0 & 0 & I & K_z \end{bmatrix}$$

while Δx (unknown terms) and Δy (input vector) are defined as

$$\Delta x = \begin{bmatrix} \Delta u \\ \Delta q \\ \Delta\lambda \\ \Delta\tau \\ \Delta\sigma \\ \Delta z \end{bmatrix} \quad \Delta y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ K_z \Delta z_r \end{bmatrix}.$$

The solution of this linear system allows us to find a mapping between the input controlled variable, i.e., the synergy reference variation Δz_r, and the output variables. In this paper, in particular, we are interested in the study of internal force variation $\Delta\lambda$, object motion Δu, and hand configuration variation Δq.

Here, we summarize the main results and input/output relationships that will be necessary to discuss the controllability of internal forces and object motion in the following section. The solution of the linear system is detailed in the Appendix. By defining the following matrices:

$$\begin{aligned} G_K^+ &= K_s G^{\mathrm{T}}(GK_sG^{\mathrm{T}})^{-1} \\ P_q &= (I - G_K^+ G)K_s J \\ V_q &= (GK_sG^{\mathrm{T}})^{-1} GK_s J \\ U_q &= (J^{\mathrm{T}}P_q + K_{J,q} + K_{J,u}V_q) \\ X &= (U_q + K_q)^{-1}K_q \\ Z &= (S^{\mathrm{T}}U_q X S + K_{S,z}) \\ Y &= (Z + K_z)^{-1}K_z \end{aligned}$$

the solution of the system described in (17) can be expressed as

$$\Delta z = Y\Delta z_r \tag{18}$$

$$\Delta\sigma = ZY\Delta z_r \tag{19}$$

$$\Delta q = XSY\Delta z_r \tag{20}$$

$$\Delta\tau = U_q XSY\Delta z_r \tag{21}$$

$$\Delta u = V_q XSY\Delta z_r \tag{22}$$

$$\Delta\lambda = P_q XSY\Delta z_r. \tag{23}$$

Equations (20)–(23) can be simplified as

$$\Delta q = XSY\Delta z_r \tag{24}$$

$$\Delta u = V\Delta z_r \tag{25}$$

$$\Delta\lambda = P\Delta z_r \tag{26}$$

where $Q = XSY$, $P = P_q XS$, and $V = V_q XSY$ are matrices mapping synergy references values to joint configuration, contact force, and object configuration variations, respectively.

Remark 1: Starting from a reference configuration and acting on synergies Δz_r, the joint displacements depends both on the synergy matrix S and on the whole system compliance. When the hand is making contact with an object or with the environment, $\Delta q \neq S\Delta z_r$.

If the synergy actuation is perfectly stiff, i.e., if $C_z = 0$, it is easy to show that $Y = I$, and thus, $\Delta z = \Delta z_r$. Furthermore, if the links are perfectly stiff and the joint control gains are infinite, i.e., $C_q = 0$, it results that $X = I$ and $\Delta q = S\Delta z$. Summarizing, in the case of $C_z = 0$ and $C_q = 0$, one gets a simplified version of (24)

$$\Delta q = S\Delta z = S\Delta z_r \tag{27}$$

which is similar to the definition of synergy control given in [13] and [21].

B. Controllable Internal Forces

In (26), $\Delta\lambda$ corresponds to the contact force variation obtained by applying a variation on the reference synergy variables Δz_r, without modifying the external wrench w_0. These contact forces can be referred to as *controllable internal forces*: controllable, since they can be modified by acting on Δz_r and internal because they do not involve a variation in the external wrench applied on the object [1].

The control of internal forces is paramount in robotic grasping [53]. It allows steering of the contact forces to satisfy the constraints imposed by friction at the contacts, thus guaranteeing adhesion with the object is not lost, which would compromise the whole grasp.

From (26), we can define a basis matrix E_s for the subspace of controllable internal forces by postural synergies as

$$\mathcal{R}(E_s) = \mathcal{R}(P). \tag{28}$$

All internal forces controllable by synergy actions can then be parameterized through a free vector α, i.e.,

$$\Delta\lambda = E_s\alpha.$$

C. Controllable Rigid-Body Object Motions and Hand Joint Redundant Motions

Equation (25) shows how the object displacements Δu are controlled from one equilibrium configuration to another by small synergy variations Δz_r. Among all the possible motions of the grasped objects, *rigid-body motions* are paramount since they do not involve viscoelastic deformations in the contact points.

A rigid-body motion is characterized by a null variation of the contact force $\Delta\lambda$, and then, from (9), the following constraint equation holds:

$$J\Delta q - G^{\mathrm{T}}\Delta u = 0 \tag{29}$$

which relates joint displacements and object displacement.

We then need to evaluate which object rigid-body motions, complying with (29), are controllable acting on synergies. We observe that the synergy reference values that modify hand and object configuration without modifying the contact force values, from (26), are a solution of the homogeneous system

$$P\Delta z_r = 0 \tag{30}$$

in other terms, rigid-body motions are generated by reference synergy variations Δz_{rh} that belong to the P matrix nullspace

$$\Delta z_{rh} \in \mathcal{N}(P). \tag{31}$$

The corresponding object displacement, Δu_h, according to (25) is given by

$$\Delta u_h = V\Delta z_{rh}. \tag{32}$$

Furthermore, the corresponding hand configuration variation Δq_h, according to (24) is described by the vector

$$\Delta q_h = Q\Delta z_{rh}. \tag{33}$$

We observe that the synergy reference values defined in the subspace

$$\Delta z_{rr} \in (\mathcal{N}(P) \cap \mathcal{N}(V)) \tag{34}$$

i.e., the solutions of the homogeneous contact force problem defined in (30), that belong to the nullspace of V matrix, neither produce contact force variation nor object motion. They then modify hand configuration without modifying object conditions and can be referred to as *hand redundant motions*. The corresponding hand configuration variation is

$$\Delta q_{rr} = Q\Delta z_{rr}. \tag{35}$$

It is clear from (29) that hand and object configurations variations that do not involve contact force modifications can be evaluated by computing

$$\mathcal{N}\begin{bmatrix} J & -G^{\mathrm{T}} \end{bmatrix} \tag{36}$$

as described, for instance, in [48]. A matrix Γ can then be defined, whose columns form a basis of such subspace. Under the hypothesis that the object motion is not indeterminate [53], i.e., $\mathcal{N}(G^{\mathrm{T}}) \neq 0$, i.e., the object is completely restrained by contacts, matrix Γ can be expressed as

$$\Gamma = \mathcal{N}\left(\begin{bmatrix} J & -G^T \end{bmatrix}\right) = \begin{bmatrix} \Gamma_{qr} & \Gamma_{qc} \\ 0 & \Gamma_{ucq} \end{bmatrix} \tag{37}$$

where Γ_{qr} is a basis matrix of the subspace of redundant motions $\mathcal{N}(J)$, and Γ_{qc} and Γ_{ucq} are conformal partitions of a complementary basis matrix. The image spaces of Γ_{qc} and Γ_{ucq} consist of coordinated rigid-body motions of the mechanism, for the hand configuration and the object position and orientation, respectively.

The description of rigid-body motion in (37) does not take into account the synergy actuation system, and then, the solution found with this method could be infeasible acting on synergies. The rigid-body motions compatible with object equilibrium equation and reachable acting on the synergy reference values are given by

$$\Delta u_{rb} \in \mathcal{R}(\Gamma_{ucs}) = (\mathcal{R}(V) \cap \mathcal{R}(\Gamma_{ucq})). \tag{38}$$

It is worth noting that rigid-body motions of the object are not all the possible motions of the object controlled by synergies as in (25), since the subspace of all synergy controlled object motion $\mathcal{R}(V)$ also contains motions due to deformations of elastic elements in the model. Summarizing, all rigid-body displacements of the object can be parameterized through a free vector β as

$$\Delta u_h = \Gamma_{ucs}\beta.$$

Similarly, the description of hand joint redundant motions obtained from (37) do not take into account the synergy actuation system, and then, also in this case, the solution found with this method could be unfeasible acting on the synergies. The hand joint redundant motions Δq_{rr}, reachable by acting on the synergy reference values Δz_r, belong to the subspace Γ_{qrs} defined as

$$\Delta q_{rr} \in \mathcal{R}(\Gamma_{qrs}) = (\mathcal{R}(Q) \cap \mathcal{R}(\Gamma_{qr})) . \tag{39}$$

Summarizing, all hand redundant motions can be parameterized through a free vector γ as

$$\Delta q_{rr} = \Gamma_{qrs}\gamma.$$

IV. Internal Forces and Rigid-Body Motion Control

In grasps by hands controlled with few synergies, it is possible that not all the object motions and contact forces are controllable by synergistic actions. According to (38), desired quasi-static rigid-body object motions $\Delta u_{\rm des}$ can be performed if they remain within subspace $\mathcal{R}(\Gamma_{ucs})$ and analogously, according to (26) and (28), arbitrary quasi-static contact force displacements $\Delta\lambda_{\rm des}$ can be performed if they evolve within subspace $\mathcal{R}(E_s)$ defined in (28). It is worth noting that with the results obtained up to here, we can arbitrarily control motions in $\mathcal{R}(\Gamma_{ucs})$ or contact forces in $\mathcal{R}(E_s)$, but we are not guaranteed that in coupled motion and force control, we can *jointly* but *independently* control two vectors lying on these subspaces.

In grasping, however, due to the presence of unilateral conic contact constraints, task specifications cannot be given disjointly in terms of either object positions or contact forces. Therefore, conditions $\Delta u_{\rm des} \in \mathcal{R}(\Gamma_{ucs})$ and $\Delta\lambda_{\rm des} \in \mathcal{R}(E_s)$ are only necessary, but no longer sufficient, for joint control of object motions and contact forces. Moreover, specifications of jointly controllable object motions and contact forces may not exhaust the control capabilities of synergy actions for the given grasp due to the presence of synergy redundancy.

Our goal is to define a set of controlled outputs for a grasp with synergies that is guaranteed to be feasible with synergy actions, that fully exploits the control inputs, and that is convenient for the specification of the tasks. The first requirement implies that the output vector of forces and motions can be controlled by synergies, the second that controlled output vector has the same dimension n_z of the synergy vector z_r, and the third that the output vector consider the following typical approach of a grasping task:

1) contact forces that can be controlled so as to avoid violation of contact constraints;
2) object trajectories that can be accommodated for by the grasp with synergies;
3) reconfiguration of limbs in the presence of redundancy in synergies. The following theorem proposes a set of outputs for grasps with synergies.

Theorem 1: Under the technical assumption that the grasp is not indeterminate ($\mathcal{N}(G^{\rm T}) = 0$), consider the quasi-static model of any grasp with synergies described in (25) and (26). It is always possible to control, jointly but independently, the controllable internal forces, the rigid-body object motions and redundancy with the synergy displacement Δz_r control input. In the general case, since the system is compliant, a variation in the contact forces cause a displacement of the grasped object. Therefore, in general, contact force variation and object displacement are related. This theorem states that, if we have a sufficient number of DoFs, we can use them to control independently internal forces *and* rigid-body object motions. Thus, in this context, *jointly and independently* means that contact forces and object motions are not related to each other and can be independently controlled acting on the available inputs. Algebraically, this corresponds to state that for any α, β, and γ, there always exists a Δz_r solving the linear system of equations

$$\begin{bmatrix} E_s\alpha \\ \Gamma_{ucs}\beta \\ \Gamma_{qrs}\gamma \end{bmatrix} = \begin{bmatrix} P \\ V \\ Q \end{bmatrix} \Delta z_r \tag{40}$$

where Γ_{ucs} and Γ_{qr} have been defined in (37), and E_s has been defined in (28).

Moreover, solution for Δz_r is unique and the number of synergies n_z is equal to the sum of the dimensions of the controlled output subspaces as follows:

$$n_z = \#(E_s) + \#(\Gamma_{ucs}) + \#(\Gamma_{qrs}). \tag{41}$$

Proof: The theorem and the proof were originally presented in [51], however, it has been summarized here for the readers' convenience, since several additional considerations on hand model, discussed in the previous sections, have been proposed with respect to that work.

Linear system (40) can be rewritten as

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} E_s^+ P \\ \Gamma_{ucs}^+ V \\ \Gamma_{qrs}^+ Q \end{bmatrix} \Delta z_r \tag{42}$$

where $B^+ = (B^{\rm T} B)^{-1} B^{\rm T}$ denotes the pseudoinverse of a basis matrix B.

The linear system in (42) is square if the number $\#(\Gamma_{ucs}) + \#(E_s) + \#(\Gamma_{zr}) = n_z$, where $\#(N)$ denotes the number of columns of matrix N. This can be proved by observing that, since Γ_{zr}, Γ_{ucs}, and E_s are full column rank by definition, observing that $\#E_s = \#\mathcal{R}(P)$, according to (28), and $\#\mathcal{R}(\Gamma_{ucs}) + \#\mathcal{R}(\Gamma_{qr}) = \#\mathcal{N}(P)$, according to (38) and (39).

To complete the proof it suffices to show that the coefficient matrix in (42) is full row rank, which is equivalent to proving that

$$\mathcal{N}\left(\begin{bmatrix} E_s^+ P \\ \Gamma_{ucs}^+ V \\ \Gamma_{zr}^+ Q \end{bmatrix}^{\rm T}\right) = \{0\}. \tag{43}$$

Each block of the matrix in the aforementioned equation is full column rank, in fact

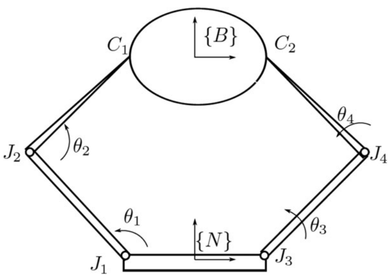


Fig. 2. Four-DoF planar gripper grasping an object with two contact points.

1) E_s is a basis for $\mathcal{R}(P)$ [cf., (28)];
2) $\mathcal{R}(\Gamma_{ucs}) \subseteq \mathcal{R}(V)$, directly from (38) and is a basis for object rigid-body motions;
3) $\Gamma_{zr} \subseteq \mathcal{R}(Q)$ from (39) and is a basis for hand redundant motions.

Hence, to prove (43), it is sufficient to show that the raw spaces of the three blocks are also mutually linearly independent and this directly follows from the definitions of internal forces, rigid-body object motions, and redundant hand motions in (28), (38), and (39). □

Remark 2: The result in (41) deals with dimensions of subspaces and is numerical in nature. It states a very basic structural property of grasp analysis with postural synergies: if n_z control inputs are available, one cannot control, jointly and independently, more than n_z variables among internal forces, object motion directions, and kinematic redundancy.

Remark 3: The result in (40) deals with grasp control with postural synergies. When the mechanical structure is complex, with many joints, but the control inputs are few, it is not easy to understand which synergy one needs to activate to accomplish a given tasks. The solution of the linear system in (40) allows us to simply compute the control variables, the synergy references, according to the task to be performed.

These results are useful also to find the minimal design requirements in terms of number of synergies to be used to accomplish the given task. It is worth emphasizing that the motions of the object considered in this paper are those performed with respect to the palm of the hand. In other terms, we are considering fine motion control of grasped objects more than large displacements, which can be performed by moving the wrist of the arm. Finally, note that results presented in this paper are still valid for fully actuated robotic hands provided the matrix S is substituted with the identity matrix.

V. Numerical Examples

The numerical simulations presented in this section were performed using Syngrasp [39], a set of MATLAB functions devoted to the simulation and analysis of the main properties of grasps performed with synergy actuated hands.

A. Simple Gripper

As a first example, let us consider a simple gripper like the one shown in Fig. 2. The gripper is planar and has two fingers, each of which is composed of two phalanges with the same lengths: the gripper has then four DoFs in total. Let $J_1, \ldots, J_4$ be the traces of the joint axes on the gripper plane, and let $\theta_1, \ldots, \theta_4$ be the joint angles. The gripper is grasping an object with its fingertips. The contact points are C_1 and C_2, the origin of the local object reference frame is on the mean point of C_1C_2 segment, and the local x-axis is parallel to C_1C_2 direction. The contact model assumed in this test is the PCWF [44], often indicated also as hard finger (HF) [53]. Then, each contact force has two components, i.e., $\lambda_1 = [\lambda_{1x} \;\; \lambda_{1y}]^{\mathrm{T}}$ and $\lambda_2 = [\lambda_{2x} \;\; \lambda_{2y}]^{\mathrm{T}}$, defined w.r.t. the object fixed reference frame. The object displacement is defined w.r.t. the base reference system by the vector $u = [u_x \;\; u_y \;\; \phi]^{\mathrm{T}}$, where ϕ represents the angle between the local and the base x-axes. We consider a reference configuration in which the external load w_0 is balanced by the contact forces λ_{01} and λ_{02}, applied at the points C_1 and C_2, respectively: the contact vector is then defined as $\lambda_0 = [\lambda_{01x} \;\; \lambda_{01y} \;\; \lambda_{02x} \;\; \lambda_{02y}]^{\mathrm{T}}$.

Indicating with a the length of the finger phalanges, the hand Jacobian matrix is defined as follows:

$$J = \begin{bmatrix} J_{1,1} & J_{1,2} & 0 & 0 \\ J_{2,1} & J_{2,2} & 0 & 0 \\ 0 & 0 & J_{3,3} & J_{3,4} \\ 0 & 0 & J_{4,3} & J_{4,4} \end{bmatrix}$$

in which the matrix terms can be expressed as

$$\begin{aligned}
J_{1,1} &= s\phi(ac_{12} + ac_1) - c\phi(as_{12} + as_1) \\
J_{1,2} &= ac_{12}s\phi - as_{12}c\phi \\
J_{2,1} &= s\phi(as_{12} + as_1) + c\phi(ac_{12} + ac_1) \\
J_{2,2} &= as_{12}s\phi + ac_{12}c\phi \\
J_{3,3} &= s\phi(ac_{34} + ac_3) - c\phi(as_{34} + as_3) \\
J_{3,4} &= ac_{34}s\phi - as_{34}c\phi \\
J_{4,3} &= s\phi(as_{34} + as_3) + c\phi(ac_{34} + ac_3) \\
J_{4,4} &= as_{34}s\phi + ac_{34}c\phi
\end{aligned}$$

with $s_1 = \sin\theta_1$, $c_1 = \cos\theta_1$, $s_{12} = \sin(\theta_1 + \theta_2)$, and so on. The grasp matrix is given by

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -r & 0 & r \end{bmatrix}$$

where r represents object radius, i.e., the distance between each contact point and the object frame origin.

The geometric terms, that express the variation of J matrix, with respect to q and u, are considered by defining the matrices $K_{J,q}$ and $K_{J,u}$ as outlined in (14). The matrix $K_{J,q}$ is given by

$$K_{J,q} = \begin{bmatrix} k_{J,q,1,1} & k_{J,q,1,1} & 0 & 0 \\ k_{J,q,2,1} & k_{J,q,2,2} & 0 & 0 \\ 0 & 0 & k_{J,q,3,3} & k_{J,q,3,4} \\ 0 & 0 & k_{J,q,4,3} & k_{J,q,4,4} \end{bmatrix}$$

in which the matrix terms can be expressed as

$$\begin{aligned}
k_{J,q,1,1} = {} & -\lambda_{01,x}(c\phi(ac_{12} + ac_1) + s\phi(as_{12} + as_1)) \\
& - \lambda_{01,y}(ac_{12}c\phi + as_{12}s\phi)
\end{aligned}$$

$$\begin{aligned}
k_{J,q,1,2} &= -\lambda_{01,x}(c\phi ac_{12} + s\phi as_{12}) - \lambda_{01,y}(ac_{12}c\phi + as_{12}s\phi)\\
k_{J,q,2,1} &= \lambda_{01,x}(s\phi(ac_{12} + ac_1) - c\phi(as_{12} + as_1))\\
&\quad + \lambda_{01,y}(ac_{12}s\phi - as_{12}s\phi)\\
k_{J,q,2,2} &= \lambda_{01,x}(ac_{12}s\phi - as_{12}c\phi) + \lambda_{01,y}(ac_{12}s\phi - as_{12}c\phi)\\
k_{J,q,3,3} &= -\lambda_{02,x}(c\phi(ac_{34} + ac_3) + s\phi(as_{34} + as_3))\\
&\quad - \lambda_{02,y}(ac_{34}c\phi + as_{34}s\phi)\\
k_{J,q,3,4} &= -\lambda_{02,x}(c\phi ac_{34} + s\phi as_{34}) - \lambda_{02,y}(ac_{34}c\phi + as_{34}s\phi)\\
k_{J,q,4,3} &= \lambda_{02,x}(s\phi(ac_{34} + ac_3) - c\phi(as_{34} + as_3))\\
&\quad + \lambda_{02,y}(ac_{34}s\phi - as_{34}s\phi)\\
k_{J,q,4,4} &= \lambda_{02,x}(ac_{34}s\phi - as_{34}c\phi) + \lambda_{02,y}(ac_{34}s\phi - as_{34}c\phi)
\end{aligned}$$

while the matrix that expresses the hand Jacobian derivatives with respect to object displacement is given by

$$K_{J,u} = \begin{bmatrix} 0 & 0 & k_{J,u,1,3}\\ 0 & 0 & k_{J,u,2,3}\\ 0 & 0 & k_{J,u,3,3}\\ 0 & 0 & k_{J,u,4,3} \end{bmatrix}$$

in which the matrix terms can be expressed as

$$\begin{aligned}
k_{J,u,1,3} &= \lambda_{01,x}(c\phi(ac_{12} + ac_1) + s\phi(as_{12} + as_1))\\
&\quad + \lambda_{01,y}(ac_{12}c\phi + as_{12}s\phi)\\
k_{J,u,2,3} &= -\lambda_{01,x}(s\phi(ac_{12} + ac_1) - c\phi(as_{12} + as_1))\\
&\quad - \lambda_{01,y}(ac_{12}s\phi - as_{12}c\phi)\\
k_{J,u,3,3} &= \lambda_{02,x}(\mathbf{c}\phi(ac_{34} + ac_3) + s\phi(as_{34} + as_3))\\
&\quad + \lambda_{02,y}(ac_{34}c\phi + as_{34}s\phi)\\
k_{J,u,4,3} &= \lambda_{02,x}(\mathbf{c}\phi(ac_{34} + ac_3) + s\phi(as_{34}\\
&\quad + as_3)) + \lambda_{02,y}(ac_{34}c\phi + as_{34}s\phi).
\end{aligned}$$

In the numerical simulations that follow, we assume that the reference configuration is described by these quantities:

1) $\theta_1 = \frac{3}{4}\pi$ rad, $\theta_2 = -\frac{\pi}{2}$ rad, $\theta_3 = \frac{\pi}{4}$ rad, $\theta_4 = \frac{\pi}{2}$ rad;
2) $w_0 = 0$ N, $\lambda_{01} = [1, 0]^{\mathrm{T}}$ N, $\lambda_{02} = [-1, 0]^{\mathrm{T}}$ N;
3) $a = 0.3$ m.

The stiffness matrices are $K_s = k_s I_{4,4}$ and $K_q = k_q I_{4,4}$, where $k_s = 1000$ N/m, $k_q = 1000$ Nm/rad, and $I_{4,4}$ represents the 4-D diagonal matrix. In this simple example, we suppose to control each joint independently, which implies $S = I_{4,4}$, and we ignore the synergy stiffness.

According to (26), in the previous described reference configuration, we obtain the following matrix P that maps input joint references to contact forces:

$$P = \begin{bmatrix} -67.44 & -33.95 & 67.25 & 33.58\\ 0.00 & 0.00 & 0.00 & 0.00\\ 67.44 & 33.95 & -67.25 & -33.58\\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}.$$

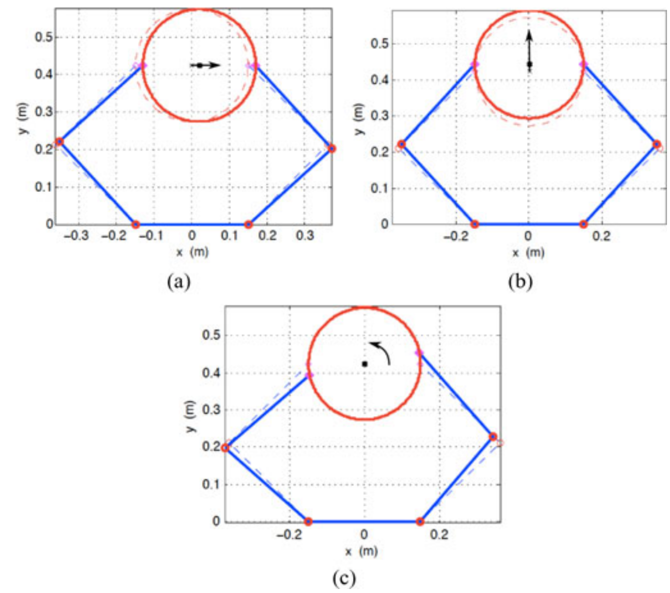


Fig. 3. Four-DoF planar gripper example: a basis for object rigid-body motions. The rigid-body motion subspace has dimension three. The dashed lines represent the initial reference position, the solid ones represent the modified configuration, the black arrows show object motion directions. (a) Translation in the x-direction. (b) Translation in the y-direction. (c) Object rotation.

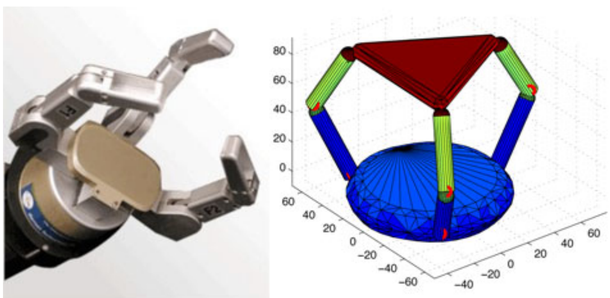
It easy to verify that in this case $\#(P) = 1$ and a basis of P image is

$$E = \begin{bmatrix} -0.71\\ 0.00\\ 0.71\\ 0.00 \end{bmatrix}$$

that consists of two forces whose direction is along the x-axis of the object reference frame, with the same moduli and opposite signs. Consequently, $\#\mathcal{N}(P) = 3$, and thus, the rigid-body object motions compatible with the contact constraints, that do not involve contact force variations can be described as a generic translation and rotation of the object on the plane, as shown in Fig. 3.

A basis of the rigid-body motions compatible with hand kinematics and the actuation system can be calculated also as the intersection between the object motions that do not produce contact force variations, i.e., that belong to $\mathcal{N}\left(\left[J - G^{\mathrm{T}}\right]\right)$, and the object motion that do not produce external load variation, i.e., that belong to $\mathcal{N}(G)$; in the reference configuration previously described, from (37) we have

$$\Gamma = \mathcal{N}\left(\left[J - G^{\mathrm{T}}\right]\right) = \begin{bmatrix} -0.77 & 0.00 & 0.00\\ 0.26 & 0.76 & 0.00\\ -0.51 & 0.44 & 0.38\\ -0.26 & -0.11 & -0.76\\ 0.09 & -0.05 & 0.00\\ 0.02 & 0.03 & 0.03\\ 0.00 & -0.46 & 0.53 \end{bmatrix}$$



(a) (b)

Fig. 4. Barrett hand. (a) Robotic hand. (b) Mathematical model representation, including the hand links, joints, base, grasped object and contact points.

TABLE I
PARAMETERS FOR THE BARRETT HAND EXAMPLE. (a) DENAVIT HARTENBERG PARAMETERS. (b) JOINT ANGLES IN THE REFERENCE CONFIGURATION

link	α_i	a_i	θ_i	d_i
finger 1				
1	0	a_1	$\theta_{1,1}$	0
2	0	a_2	$\theta_{1,2}$	0
finger 2				
1	0	0	$\theta_{2,0}$	0
2	$\pi/2$	a_1	$\theta_{2,1}$	0
3	0	a_2	$\theta_{2,2}$	0
finger 3				
1	0	0	$\theta_{3,0}$	0
2	$\pi/2$	a_1	$\theta_{3,1}$	0
3	0	a_2	$\theta_{3,2}$	0

(a)

θ_i	value (rad)
$\theta_{1,1}$	$\pi/3$
$\theta_{1,2}$	$\pi/3$
$\theta_{2,0}$	$2\pi/3$
$\theta_{2,1}$	$\pi/3$
$\theta_{2,2}$	$\pi/3$
$\theta_{3,0}$	$-2\pi/3$
$\theta_{3,1}$	$\pi/3$
$\theta_{3,2}$	$\pi/3$

(b)

and then the object body motions that do not modify contact forces are in the subspace defined by

$$\Gamma_{ucq} = \begin{bmatrix} 0.09 & -0.05 & 0.00 \\ 0.02 & 0.03 & 0.03 \\ 0.00 & -0.46 & 0.53 \end{bmatrix}.$$

It is easy to note that this subspace is equivalent to those evaluated as $\mathcal{N}(P)$ and corresponds to a generic translation and rotation of the object in the plane.

B. Barrett Hand

Results on the motion and force control of grasps with synergies have been then applied to a robotic hand whose kinematics was inspired by the Barrett hand (Barrett Technology, Inc.), shown in Fig. 4. It is a three finger, eight-axis mechanical hand, in which each finger has two joints. One of the fingers, referred to as 1, is stationary, while the other two can spread synchronously up to 180 degrees about the palm. Although there are eight axes, the hand is actuated by four motors: each finger has an actuated proximal link and a coupled distal link that moves at a fixed rate with the inner link. An additional motor controls the synchronous spread of the two fingers about the palm. In the Barrett hand, a clutch mechanism allows the outer link to continue to move even if the inner link motion is obstructed, however, this feature has not been considered in the present analysis.

Let us define $\theta_{i,1}$ $(i = 1, \ldots, 3)$ the rotation of the inner link with respect to the palm, $\theta_{i,2}$ $(i = 1, \ldots, 3)$ the rotation of the outer link with respect to the inner one, and $\theta_{i,0}$ $(i = 2, 3)$ the spread of the two fingers about the palm. Thus, the configuration vector can be defined as: $q = [\theta_{1,1}, \theta_{1,2}, \theta_{2,0}, \theta_{2,1}, \theta_{2,2}, \theta_{3,0}, \theta_{3,1}, \theta_{3,2}]^T$. The Denavit Hartenberg parameters for the Barrett hand are summarized in Table I(a). In the numerical simulations, we assumed $a_1 = a_2 = 0.05$ m.

The mechanical couplings between the joints are expressed by the following relationships:

$$\theta_{2,0} = -\theta_{3,0} = z_1$$
$$\theta_{i,2} = \alpha_i \theta_{i,1} = z_{i+1} \quad i = 1, \ldots, 3$$

where α_i represents the ratio between the outer and the inner angle for the ith link. In the numerical simulation described previously, we assumed $\alpha_i = 1$. The joint angles are controlled acting on four parameters, collected in the vector $z = [z_1, \ldots, z_4]^T$. Accordingly, the synergy matrix can be defined as

$$S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \alpha_2 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \alpha_3 \end{bmatrix}. \tag{44}$$

In this example, the contact stiffness matrix has been chosen as $K_s = k_s I_9$, where $k_s = 1000$ N/m and I_9 is the 9×9 identity matrix. The joint stiffness matrix has been chosen as $K_q = k_q I_8$, where $k_q = 1000$ Nm/rad and I_8 is the 8-D identity matrix. Finally, the synergy stiffness matrix has been chosen as $K_z = k_z I_8$, where $k_z = 1000$ Nm/rad and I_8 is the 8×8 identity matrix. The initial contact force λ_0 has been considered zero so that the components of the geometric terms $K_{J,q}$ and $K_{J,u}$ vanish.

Reference values for the hand joints are summarized in Table I(b). The contact points between the hand and the grasped object were located on the three finger-tips and the normal directions at the contact points have been thought as oriented toward the center of the object. The HF contact model has been considered in this example.

Hand Jacobian matrix J dimensions are 9×8, matrix G dimensions are 6×9, and the $\mathcal{N}(G)$ dimension is 3. According to the previously described analysis, the dimension of the controllable internal forces and object motions have been evaluated with both the hypothesis that the hand is controlled with the four actuators, and considering the case when all the eight joints are actuated. Table II summarizes the obtained results, in particular, the dimensions of the controllable internal forces, rigid-body motions, and hand redundancy subspaces. We can observe that, in all the cases, the sum between the dimensions of E_s, Γ_{ucs}, and Γ_{zr} is equal to the number of synergies that is to the number of actuated joints.

TABLE II
BARRETT HAND: CONTROLLABLE INTERNAL FORCES AND ALLOWABLE MOVEMENTS FOR THE SYNERGY ACTUATED (FOUR DOFS) AND FULLY ACTUATED (EIGHT DOFS) HAND

	E	Γ_{ucs}	Γ_{zr}
synergies (4 inputs)	3	1	0
fully actuated (8 joints)	3	2	3

In particular, for the synergy actuated case, the following values were obtained for matrices P and V:

$$P = \begin{bmatrix} 0.17 & 0.42 & 0.04 & 0.04 \\ 0 & 0 & 0.28 & -0.28 \\ 0 & 0 & 0 & 0 \\ -0.09 & -0.21 & -0.26 & 0.22 \\ 0.15 & 0.08 & -0.39 & -0.12 \\ 0 & 0 & 0 & 0 \\ -0.09 & -0.21 & 0.22 & -0.26 \\ -0.15 & .08 & 0.12 & 0.39 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.44 & 0.19 & -0.10 & -0.10 \\ 0 & 0 & -0.02 & 0.02 \\ 0 & 0.01 & 0.01 & 0.01 \\ 0 & 0 & -0.05 & 0.05 \\ 0.15 & -0.11 & 0.06 & 0.06 \\ 0 & 0 & 0.05 & -0.05 \end{bmatrix}.$$

It is possible to verify that in this case, $\#(P) = 3$, and a basis of the controllable internal forces is given by

$$E_s = \begin{bmatrix} 0.06 & 0.08 & 0.81 \\ -0.46 & 0.35 & 0 \\ 0 & 0 & 0 \\ 0.37 & -0.34 & -0.41 \\ -0.19 & -0.73 & 0.09 \\ 0 & 0 & 0 \\ -0.43 & 0.26 & -0.40 \\ 0.65 & 0.39 & -0.09 \\ 0 & 0 & 0 \end{bmatrix}.$$

Fig. 5 shows, for the first three synergies, the hand motion (first row) and the set of internal contact forces generated acting through each single synergy, evaluated by means of (26) (second row). Only three synergies have been represented since the activation of the fourth synergy is symmetric with respect to the third one.

C. Application to a Human-Like Robotic Hand

The analysis of controllable internal forces and object movements has been applied to a robotic hand with an anthropomorphic kinematic structure, actuated with synergies. The details of the kinematic model are described in [42] and [43]. The kinematic model of the anthropomorphic hand considered has globally 20 DoFs, four DoFs for each finger. In this paper, a tripod grasp has been considered. The object (a cherry) is grasped with the thumb, index, and middle finger. Each of the three fingers touches the object only in its tip. An HF contact model has been considered (single PCWF). The layout of the hand and the object is shown in Fig. 6.

The contact force and joint vector dimensions are then $n_l = 9$ and $n_q = 20$, respectively. Thus, for the fully actuated hand, grasp matrix and hand Jacobian dimensions are, respectively, $G \in \Re^{6\times 9}$ and $J \in \Re^{9\times 20}$. The dimension of internal force subspace is $e = \#(P) = 3$, the rank of rigid-body motion subspace is $\#(\Gamma_{uc}) = 4$, and the redundancy is $\#(\Gamma_{qr}) = 13$.

In order to reduce the number of controlled joint inputs, a synergy-based actuation system has been considered. The synergy matrix S is computed such that its columns are the principal components (PCs) extracted from the dataset presented in [54]. In this study, the authors collected a large set of data containing grasping poses from subjects that were asked to shape their hands in order to mime grasps for a large set ($N = 57$) of common objects. PCA of this data revealed that the first two PCs account for more than the 80% of the variance, suggesting that a satisfying characterization of the recorded data can be obtained using a much lower-dimensional subspace of the hand-DoF space. These and similar results seem to suggest that, out of the 20 or more DoFs of a human hand, only a few combinations can be used to shape the hand for basic grasps used in everyday life. The data were obtained using an instrumented glove that measured the configuration of 15 hand joints. From the available experimental data, the PCA returned a 15×15 matrix whose columns represented the PCs of the dataset, ordered in such a way that the first one accounts for the largest possible variance (that is, accounts for as much of the variability in the data as possible), and each succeeding component, in turn, has the highest variance possible under the constraint to be orthogonal to (i.e., uncorrelated with) the preceding components. Since the anthropomorphic model we adopted has 20 DoFs, while the measured joints were 15, we extended with some heuristic considerations, based on the human hand anatomy, the PCA results and obtained a complete synergy matrix $S_{\text{tot}} \in \Re^{20\times 15}$. From this matrix, we selected the synergy matrix $S \in \Re^{20\times n_z}$, with n_z varying from 1 to 15, selecting from S_{tot} the first n_z columns. In the preceding section, the synergies were modeled through a compliant structure. In addition, in this case, the contact stiffness matrix has been chosen as $K_s = k_s I_9$, where $k_s = 100$ N/m, and I_9 is the 9×9 identity matrix. The joint stiffness matrix has been chosen as $K_q = k_q I_{20}$, where $k_q = 1000$ Nm/rad, and I_{20} is the 20×20 identity matrix. Finally, the synergy stiffness matrix has been chosen as $K_z = k_z I_{n_z}$, where $k_z = 10000$ Nm/rad, and I_{n_z} is the $n_z \times n_z$ identity matrix. For the sake of simplicity, the initial contact force λ_0 has been considered null so that the components of the geometric terms $K_{J,q}$ and $K_{J,u}$ are null.

The number of engaged synergies has been progressively increased from 1 to 9, in the order obtained from PCA decomposition of experimental measures [51], [54].

Fig. 7 shows the internal force variation $\Delta\lambda$ and the corresponding object displacement Δu obtained activating one

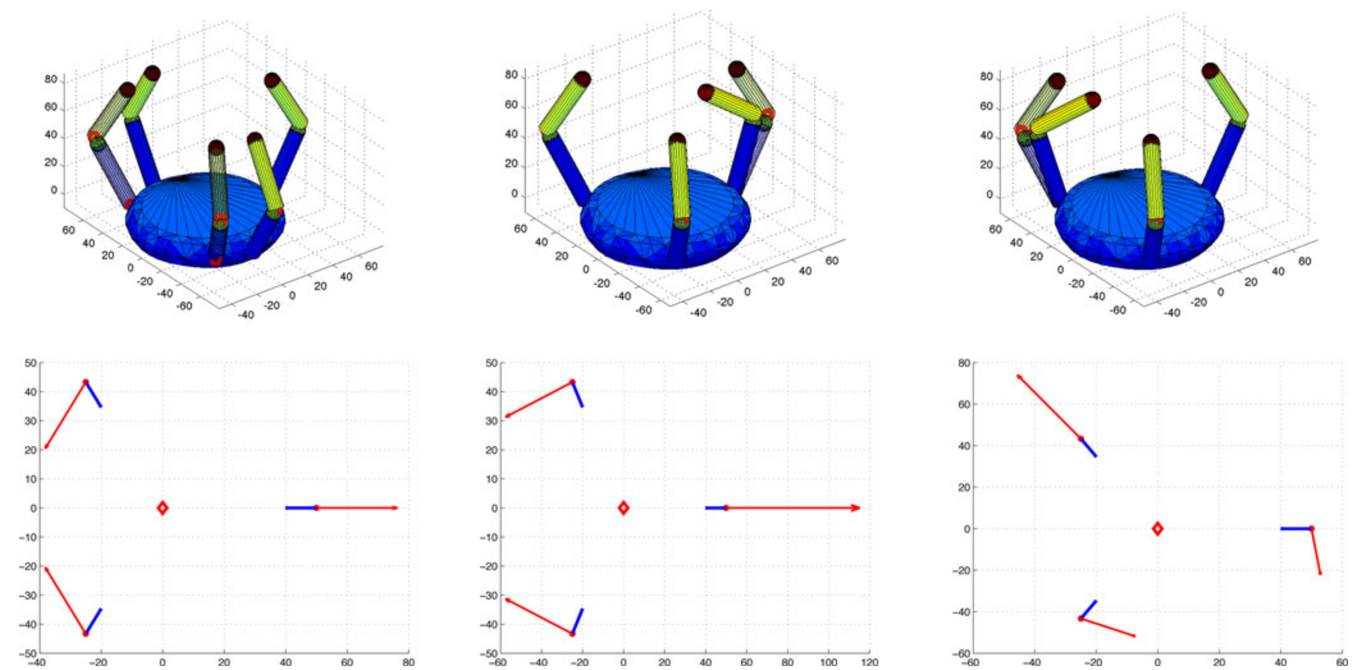


Fig. 5. Synergies in the Barrett hand. (First row) Hand configuration obtained acting on each synergy. (Second row) Contact points (red dots), object center (red square), contact normal unit vectors (blue arrows), and internal forces (red arrows) generated activating each synergy.

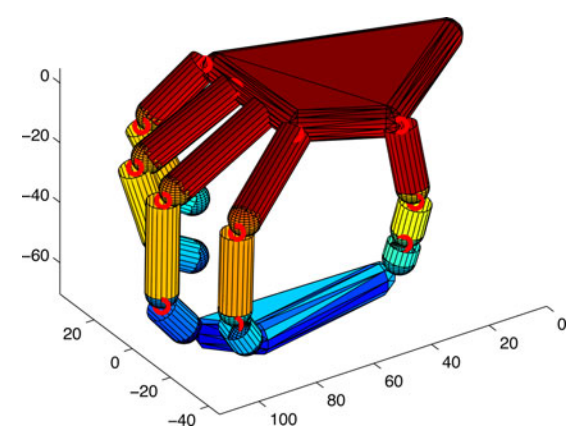


Fig. 6. Human-like hand and the grasped object models in the reference configuration.

synergy once, i.e., $\Delta\lambda_i = P\Delta z_{ri}$

$$\Delta u_i = P\Delta z_{ri}$$

where $\Delta z_{ri} = [0, 0, \ldots, 1, \ldots, 0]^{\mathrm{T}}$.

In order to verify the results presented in the preceding sections, we analyzed, for different numbers of engaged synergies, the dimensions of controllable internal forces $e = \#(P)$, rigid-body motions $\#(\Gamma_{uc})$, and redundant hand motion $\#(\Gamma_r)$. Results in terms of dimensions of controllable forces and movements subspaces, are shown in Table III. We observe that, by increasing the number of engaged synergies from 1 to 3, the number of controllable internal forces increases from 1 to 3, while no rigid motions are possible. Increasing the number of engaged synergies from 4 to 7, the dimension of controllable rigid-body motions increases from 1 to 4, while the controllable internal force subspace does not increase any more, since it dimension reached the size of $\mathcal{N}(G)$. Finally, further increasing the number of synergies, either the controllable rigid-body motion dimensions fulfill or hand redundant motions appear. From the results summarized in Table III, it is evident that the maximum dimension of the subspace of controllable object rigid motion, in this application, is $\#\Gamma_{ucs} = 4$, therefore, it is not possible to fully control the 6-D motion of the object without modifying contact forces. In particular, if for instance, only $n_z = 6$ synergies are activated, $\#\Gamma_{ucs} = 3$, and the following basis for object rigid-body motions can be found:

$$\Gamma_{ucs} = \begin{bmatrix} -4.11 & 2.76 & 4.48 \\ -10.3 & 3.08 & 10.6 \\ -3.91 & -5.45 & -8.74 \\ -0.11 & -0.12 & -0.39 \\ -0.06 & 0.12 & -0.28 \\ -0.20 & 0.09 & 0.39 \end{bmatrix}$$

that can be obtained by choosing the reference synergy values from the following base:

$$\Gamma_{zr} = \begin{bmatrix} -0.61 & 0 & 0 \\ -0.27 & -0.93 & 0 \\ -0.57 & 0.18 & 0.12 \\ -0.36 & 0.27 & 0.38 \\ 0.31 & -0.15 & 0.70 \\ 0.03 & 0.03 & -0.59 \end{bmatrix}.$$

If, for instance, we were interested only in the translational part of the motion, with no concern for rotations, we could evaluate the synergy variation necessary to produce the desired

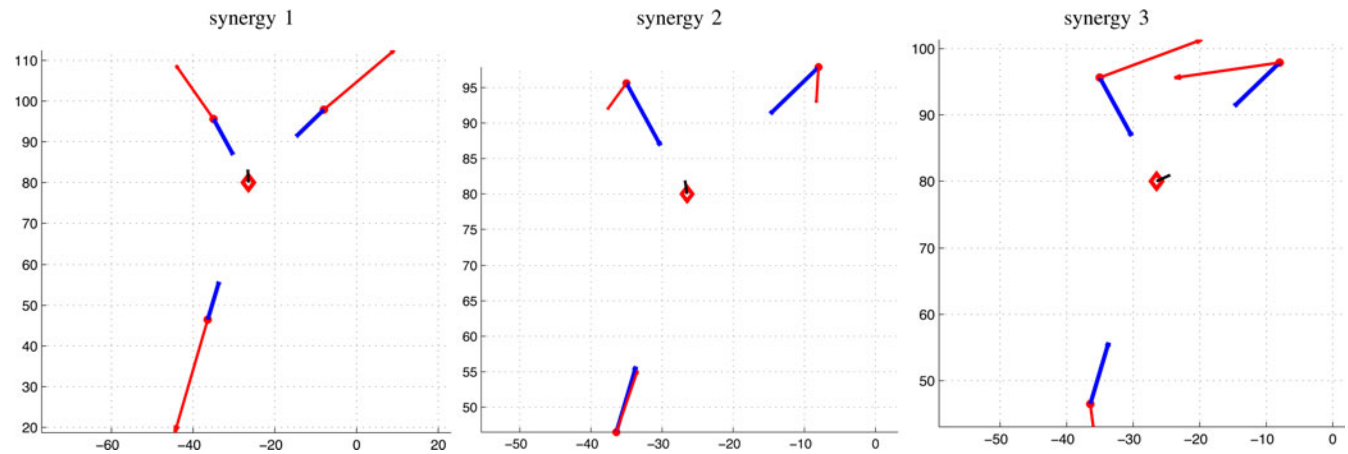


Fig. 7. Forces and movements produced by activating one synergy once. Internal forces (red arrows), contact point unitary vectors (blue arrows), and object displacement (black arrow) induced by the application of one synergy once, projected on the xy plane. The red round dots represent the contact points and the red square dot is the object center.

TABLE III
HUMAN-LIKE HAND: DIMENSION OF CONTROLLABLE INTERNAL FORCES E_s, RIGID-BODY MOTIONS Γ_{uc}, REDUNDANT MOTIONS Γ_r, AS A FUNCTION OF THE NUMBER OF ACTIVATED SYNERGIES

n_z	$\#E$	$\#\Gamma_{ucs}$	Γ_r
1	1	0	0
2	2	0	0
3	3	0	0
4	3	1	0
5	3	2	0
6	3	3	0
7	3	4	0
8	3	4	1
9	3	4	2

displacement of the object center, Δp_{des}, as follows:

$$\Delta z_r = \Gamma_{zr}\Gamma_{ucs,t}^{-1}\Delta p_{\text{des}}$$

where $\Gamma_{ucs,t}$ is obtained from the first three rows of matrix Γ_{ucs}. For example, to obtain a displacement of 1 mm in the x-, y-, and z-direction, respectively, without changing the contact forces, the following synergy reference variations have to be commanded

$$\Delta z_{rx} = [0.10, -0.63, 0.18, 0.12, -0.43, 0.24,]^{\mathrm{T}}$$
$$\Delta z_{ry} = [0, 0.27, -0.03, -0.01, 0.17, -0.12,]^{\mathrm{T}}$$
$$\Delta z_{rz} = [0.05, 0.01, 0.04, 0.02, -0.09, 0.05,]^{\mathrm{T}}.$$

These examples show how the proposed model can be used to define the synergy input variations necessary to perform a desired task that could consist of a variation in the contact forces and/or on a displacement of the grasped object with respect to the hand reference frame. Furthermore, the examples show how the model could be used in the design phase of the hand and of its control system, to define the structural properties of the hand synergy system necessary to realize predefined tasks.

VI. CONCLUSION

This paper has provided geometric and structural properties of hands actuated by a postural synergy system, that are the fundamentals for the design of control strategies for performing complex manipulation tasks, involving both control of motion and forces, through very few control inputs. In grasping hands with n_z postural synergies, a structural relationship exists between the dimension of controllable internal forces and object motion subspaces, and the number of synergy control inputs. In particular, in this paper, we verified that if the hand is actuated controlling the reference values of the synergy vector $z_r \in \Re^{n_z}$, it is not possible to control, jointly and independently, more than n_z variables among internal forces, object motion directions, and kinematic redundancy.

Furthermore, tools for design requirements of complex robotic hands in terms of number of synergies to accomplish manipulation tasks are provided. Providing structural and basic results like the controllability of forces and motions in hand grasps with postural synergies will allow us to better understand and exploit the synergies in both robotics and human studies. Some numerical examples, relative to a simple gripper, a three fingered robotic hand with a kinematic structure similar to the Barrett hand, and an anthropomorphic hand, are shown putting the theory to the test.

The results obtained from the analysis of the grasp model presented in this paper are the basis for the design of new robotic hands with a low number of actuators, based on synergistic organization of joint references, as those proposed in [15] and [16], and also for the development of new control strategies for existing robotic hands [32], [50].

APPENDIX

A. Grasp Matrix and Hand Jacobian Matrix Evaluation

In the following, we describe the evaluation of grasp matrix and hand Jacobian matrix in the 3-D case, i.e., for $n_d = 6$. The simplification for the bidimensional case (i.e., $n_d = 2$) is straightforward. Further details and examples can be found, for instance in [44] and [53]. A detailed explanation of the representation can be found in [38].

Let $g_{bc_i^o}$ be the vector that describes the configuration of frame $\{C_i^o\}$ with respect to $\{B\}$. Furthermore, let $g_{nc_i^h}$ be the vector that describes the configuration of frame $\{C_i^h\}$ with respect to $\{N\}$. We use the product of exponential formula for its parameterization, i.e.,

$$g_{nc_i^h} = \left(\prod_{k=1}^{m_i} e^{\hat{\psi}_k q_k^i}\right) g_{nc_i^h}(0)$$

where $\psi_k \in \Re^6$ is an element of $se(3)$, and $\hat{\psi}_k \in \Re^4$ is its homogeneous form, q_i^k are the exponential coordinates for a local representation of $SE(3)$, and $g_{nc_i^h}(0)$ is the initial configuration.

Grasp Matrix: Let $w_{c_i^h}^{c_i^o} \in \Re^{n_d}$ be the wrench that the hand exerts on the object on the ith contact point, whose components are expressed with respect to $\{C_i^o\}$ reference frame. The object equilibrium, with respect to $\{B\}$ reference system, can be expressed as

$$w + \sum_{i=1}^{n_c} \mathrm{Ad}_{g_{bc_i^o}}^{-\mathrm{T}} w_{c_i^h}^{c_i^o} = 0.$$

For each contact point i, we can introduce the vector $\lambda_i \in FC_i$, where FC_i is a subspace of dimension l_i whose value depends on the type of contact. For each contact point, we can define a matrix $H_i \in \Re^{n_d \times l_i}$ that maps the local vector λ_i onto the contact wrench $w_{c_i^h}^{c_i^o}$

$$w_{c_i^h}^{c_i^o} = H_i \lambda_i.$$

The equilibrium equation can be written as

$$w + \sum_{i=1}^{n_c} \mathrm{Ad}_{g_{bc_i^o}}^{-\mathrm{T}} H_i \lambda_i = 0.$$

The contact vectors can be organized in the vector $\lambda = [\lambda_1^{\mathrm{T}}, \ldots, \lambda_{n_c}^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{n_l}$, the equilibrium equation can be furthermore simplified as

$$w + G\lambda = 0$$

where the Grasp matrix $G \in \Re^{n_d \times n_l}$ is defined as

$$G = \left[\mathrm{Ad}_{g_{bc_1^o}}^{-\mathrm{T}} H_1, \ldots, \mathrm{Ad}_{g_{bc_{n_c}^o}}^{-\mathrm{T}} H_{n_c}\right].$$

Let ξ be the twist that describes the $\{B\}$ frame motion with respect to $\{N\}$, expressed with respect to $\{B\}$ reference frame, and let $\xi_{c_1^o}^{c_i^o}$ be the twists of frames $\{C_i^o\}$, expressed with respect to $\{C_i^o\}$. These twists are related by the following relationship:

$$\xi_{ab}^{c_i^o} = \mathrm{Ad}_{g_{c_i^o b}} \xi.$$

Since a contact model has been defined, that constrains some of the relative motion components, we can highlight these components in a vector $v_{c_i^o}^{c_i^o} \in \Re^{l_i}$, by means of the transpose of the selection matrix previously defined

$$v_{c_i^o}^{c_i^o} = H_i^{\mathrm{T}} \xi_{c_1^o}^{c_i^o} = H_i^{\mathrm{T}} \mathrm{Ad}_{g_{c_i^o b}} \xi.$$

All these components can be collected in the vector

$$v_{c^o}^{c^o} = [v_{c_1^o}^{c_1^o \mathrm{T}}, \ldots, v_{c_{n_c}^o}^{c_{n_c}^o \mathrm{T}}]^{\mathrm{T}} \in \Re^{n_l}.$$

It is easy to observe that

$$\begin{aligned} v_{c^o}^{c^o} &= \left[(H_1^{\mathrm{T}} \mathrm{Ad}_{g_{c_1^o b}})^{\mathrm{T}}, \ldots, (H_{n_c}^{\mathrm{T}} \mathrm{Ad}_{g_{c_{n_c}^o b}})^{\mathrm{T}}\right]^{\mathrm{T}} \xi \\ &= G^{\mathrm{T}} \xi. \end{aligned}$$

For the sake of simplicity, in the text, we did not consider the relationship between angular speed and time derivative of angular displacements. When we multiply the aforementioned equation by the time Δt, in the rotational part, $\xi_{ab}^b \Delta t$ does not represent a coordinate variation. However, we can define a matrix $T(u) \in \Re^{n_d \times n_d}$ that allows us to express the frame twist as a function of u time derivative, i.e.,

$$\xi = T(u)\dot{u}$$

by defining the contact frame variation as $\Delta c_i^o = v_{c_i^o}^{c_i^o} \Delta t$, we can write $\Delta c^o = \tilde{G}^{\mathrm{T}} \Delta u$, where $\tilde{G}^{\mathrm{T}} = G^{\mathrm{T}} T(u)$. It is worth noting that G matrix elements do not depends on object displacement u, while $\tilde{G}$ does, since T depends on u.

Hand Jacobian Matrix: Let $\xi_n^{c_i^o}$ represent the twist of a frame instantaneously superimposed to the frame $\{N\}$ that is fixed with $\{C_i^h\}$ with components expressed with respect to $\{N\}$. This twist depends on the joint velocities $\dot{q}^i$ as follows:

$$\xi_n^{c_i^o} = {}^n J_i(q^i)\dot{q}^i$$

where ${}^n J_i(q^i)$ is the spatial Jacobian relative to the ith contact point, defined as

$${}^n J_i(q^i) = \left[\xi_1^i, \xi_2'^i, \ldots, \xi_{m_i}'^i\right]$$

with

$$\xi_j'^i = \mathrm{Ad}_{g_{1(j-1)}} \xi_j^i$$

and

$$g_{1(j-1)} = \prod_{k=1}^{j-1} e^{\hat{\xi}_k^i q_k^i}.$$

Let then $\xi_{c_i^h}^{c_i^o}$ the twists of frames $\{C_i^h\}$, expressed with respect to $\{C_i^o\}$. To find the twist $\xi_{c_i^h}^{c_i^o}$, we can use adjoint matrix obtaining

$$\xi_{c_i^h}^{c_i^o} = \mathrm{Ad}_{g_{c_i^o n}(u)} {}^n J_i(q^i)\dot{q}^i = \tilde{J}_i \dot{q}^i$$

with $\tilde{J}_i(q^i, u) = \mathrm{Ad}_{g_{c_i^o n}(u)}^n J_i(q^i)$. We can then highlight the velocity components constrained by the contact model in a vector $v_{c_i^h}^{c_i^o} \in \Re^{l_i}$, by means of the transpose of the selection matrix previously defined as follows:

$$v_{c_i^h}^{c_i^o} = H_i^{\mathrm{T}} \xi_{c_1^h}^{c_i^o} = H_i^{\mathrm{T}} \tilde{J}_i \dot{q}^i.$$

All these components can be collected in the vector

$$v_{c^h}^{c^o} = [v_{c_1^h}^{c_1^o \mathrm{T}}, \ldots, v_{c_{n_c}^h}^{c_{n_c}^o \mathrm{T}}]^{\mathrm{T}} \in \Re^{n_l}.$$

Similarly, we can collect all the joint variables in a vector

$$q = [q_1^{\mathrm{T}}, \ldots, q_F^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{n_q}.$$

The components of the contact point velocities $v_{c^h}^{c^o}$ are related to the hand joint velocities $\dot{q}$ by the following relationship:

$$v_{c^h}^{c^o} = J(q,u)\dot{q}$$

where

$$J(q,u) = \begin{bmatrix} H_1^{\mathrm{T}}\tilde{J}_1 & \cdots & 0 \\ \cdots & \cdots & \cdots \\ \cdots & 0 & H_{nc}^{\mathrm{T}}\tilde{J}_{nc} \end{bmatrix}$$

is the hand Jacobian matrix. It is worth noting that, since we expressed the contact twists with respect to the object fixed $\{C_i^o\}$ reference frames, J depends both on hand configuration q and on object displacement u.

Let the components of contact point twists on the hand, constrained by the contact model, and expressed with respect to $\{C_i^o\}$ reference frames, can be evaluated as a function of hand joint velocities as follows:

$$v_{c^h}^{c^o} = J\dot{q} \tag{45}$$

in which $J \in \Re^{n_l \times n_q}$ represents the hand Jacobian matrix. It is worth noting that the hand Jacobian matrix depends on both the hand joint configuration q and on object displacement vector u, i.e., $J = J(q,u)$.

B. System Solution

Let us consider the system composed of the equilibrium conditions described by (12), (13), and (15), the constitutive conditions described by (9), (10), and (11), and the congruence condition described by (7), and summarized in (17).

By substituting (9) into (12), we can express Δu as a function of Δq

$$GK_s\left(J\Delta q - G^{\mathrm{T}}\Delta u\right) = 0$$

$$GK_sJ\Delta q = GK_sG^{\mathrm{T}}\Delta u = 0$$

$$\Delta u = \left(GK_sG^{\mathrm{T}}\right)^{-1}GK_sJ\Delta q = V_q\Delta q \tag{46}$$

where $V_q = \left(GK_sG^{\mathrm{T}}\right)^{-1}GK_sJ$. Let us substitute (46) into (9), we can then express $\Delta\lambda$ as a function of Δq as

$$\Delta\lambda = K_s\left(J\Delta q - G^{\mathrm{T}}\left(GK_sG^{\mathrm{T}}\right)^{-1}GK_sJ\Delta q\right)$$

$$\Delta\lambda = \left(I - G_K^{+}G\right)K_sJ\Delta q = P_q\Delta q \tag{47}$$

where $G_K^{+} = K_sG^{\mathrm{T}}\left(GK_sG^{\mathrm{T}}\right)^{-1}$ is the G matrix pseudoinverse, weighted with K_s matrix and $P_q = \left(I - G_K^{+}G\right)K_sJ$. Let us then consider (13) and substitute $\Delta\lambda$ and Δu with (47) and (46), respectively, as follows:

$$\Delta\tau = \left(J^{\mathrm{T}}P_q + K_{J,q} + K_{J,u}V_q\right)\Delta q = U_q\Delta q \tag{48}$$

with $U_q = \left(J^{\mathrm{T}}P_q + K_{J,q} + K_{J,u}V_q\right)$. By substituting (48) into (10), we can express Δq as a function of Δq_r as

$$\Delta\tau = K_q\left(\Delta q_r - \Delta q\right)$$

$$U_q\Delta q = K_q\left(\Delta q_r - \Delta q\right)$$

$$\left(U_q + K_q\right)\Delta q = K_q\Delta q_r$$

$$\Delta q = \left(U_q + K_q\right)^{-1}K_q\Delta q_r = X\Delta q_r \tag{49}$$

where $X = \left(U_q + K_q\right)^{-1}K_q$. Taking into account (7), we can express Δq as a function of Δz as

$$\Delta q = XS\Delta z \tag{50}$$

and consequently, by substituting into (48), we obtain

$$\Delta\tau = U_qXS\Delta z. \tag{51}$$

By substituting (51) into (15), we can express $\Delta\sigma$ as a function of Δz as

$$\Delta\sigma = S^{\mathrm{T}}\Delta\tau + K_{S,z}\Delta z$$

$$= \left(S^{\mathrm{T}}U_qXS + K_{S,z}\right)\Delta z$$

$$\Delta\sigma = \left(S^{\mathrm{T}}U_qXS + K_{S,z}\right)\Delta z = Z\Delta z \tag{52}$$

where $Z = \left(S^{\mathrm{T}}U_qXS + K_{S,z}\right)$. Finally, by substituting (52) into (11), we can express Δz as a function of Δz_r as

$$\Delta\sigma = K_z\left(\Delta z_r - \Delta z\right)$$

$$Z\Delta z = K_z\left(\Delta z_r - \Delta z\right)$$

$$\left(Z + K_z\right)\Delta z = K_z\Delta z_r$$

$$\Delta z = \left(Z + K_z\right)^{-1}K_z\Delta z_r = Y\Delta z_r \tag{53}$$

with $Y = \left(Z + K_z\right)^{-1}K_z$. By backward substituting (53) and (50) into (47), (46), etc., we find the system solution shown in (18)–(26).

Acknowledgment

The authors would like to thank E. Farnioli for his suggestions in the development of the mathematical model and M. Santello for the inspiring discussions and for providing experimental data and parameters.

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