Evaluation of Grasp Stiffness in Underactuated Compliant Hands

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Abstract— **Underactuation represents a solution for the simplification of robotic hand structures, that allows to keep adaptability and robustness in uncertain and unstructured environments. In such systems, the unactuated Degrees of Freedom present passive elastic elements. The performance of an underactuated robotic hand depends on its mechanical properties and in particular, on the compliance of its elements. Grasp stiffness is an important parameter for characterizing the grasp, it describes a linear relationship between the forces applied in the grasp and the resulting object motion, and depends on structural compliance of hand elements and of the contact, and on the gains of actuator control systems. In this paper a quasi-static model of grasp with underactuated hands is presented and applied to the evaluation of grasp stiffness. The model is then applied to a simple underactuated planar hand, to evaluate the dependency of grasp stiffness on some of its structural properties.**

I. Introduction

Grasping and dexterous manipulation with robotic systems is challenging, especially in unstructured and uncertain environment. Robotic hands could obtain the needed adaptability by increasing the number of actuators, using tactile and visual sensor feedback and complex control and task planning algorithms, often inspired by the anthropomorphic hand structure [1]. This approach is undoubtedly optimal to obtain the desired performance, but, often optimal results comes at the cost of size, mechanical and control complexity.

Researchers attempted to simplify robotic hand structure by keeping adaptability and dexterity through reducing the number of actuators.

One possibility is the coupling between the joints reducing the overall number of Degrees of Freedom. A fixed mechanical motion coupling between hand joints is adopted in this case [2].

Another possible solution in the simplification way is to give to some joints a passive compliance, that allows to keep adaptability properties and to gain robustness with respect to uncertainties [3]. Underactuation represents a way to simplify hand complexity while keeping the ability of the hand to adapt to uncertain object shapes during grasp operation. Generally speaking, a mechanism is defined underactuated when it has more Degrees of Freedom (DoFs) than actuators [4]. Decreasing the number of actuators may reduce manipulability and controllability properties of the hand, as discussed in [5], that concerned with hands in which the joints were coupled through a compliant system, inspired by the synergy organization of human hand motion [6].

In order to maintain the versatility properties while simplifying robotic hand structure through underactuation and passive joints, theoretical tools that allows to design and optimize hand parameters are needed. In [7] the kinetostatic analysis of underactuated robotic hands is presented. The force distribution during grasping operation is analysed and tools for the investigatiion of grasp stability are provided. The form closure properties have been extended to underactuated hands in [8]. In [9] dexterous manipulation properties with underactuated elastic hands is discussed. In [10] the author discusses the problem of force isotropy in underactuated hands, this property guarantee a uniform distribution of forces over the grasped object and prevents object damages due to force unbalances.

One important parameter that has to be considered in the design of a robotic hand and, more in general, of a robotic system, is the stiffness. This parameter is particularly significant in grasp operatinos, when the hand has to apply to the object a force and impose a motion [11].

Stiffness analysis evaluates the resistance of the robot system to the deformation caused by an external load variation applied on the end effector. In grasp operation, stiffness measures the capability of the grasped object to resist to external load variations.

In a generic robot system, traditionally researchers distinguish between mechanical stiffness, or passive compliance, that is caused by the structural deformation of robot components when the system is subject to a load, and the virtual elasticity of actuators, that can be adjusted by acting on the control parameters [12].

Different methods are available for the evaluation of the mechanical stiffness of a manipulator. In this paper we will refer to the so-called Virtual Joint Method, that substantially represents the robot as a rigid multi-body system in which the links are connected by compliant joints. This technique, even if is more approximated than other methods (e.g. Finite Element Methods, FEM), is more suitable in the early design phase and in the analysis of the overall manipulator dynamic properties.

Furthermore, conventional stiffness analysis usually considers the unloaded case. The analysis of the loaded case lead to the so-called geometric stiffness terms, that depends on the manipulator configuration variation and thus on the second derivative of kinematic relationships [13],[14], [12].

Stiffness evaluation is an important aspect to be considered

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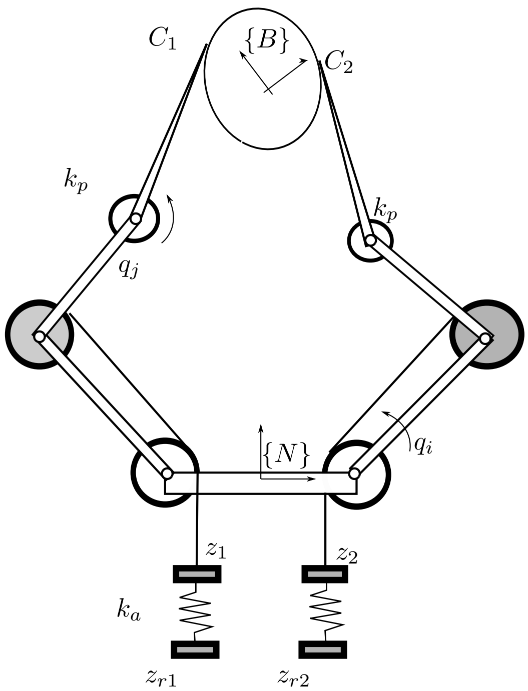

Fig. 1: An underactuated hand with passive joints, inspired from [9].

in underactuated robotic hands, in which elastic passive elements are present, and have to be properly designed [15]. This paper integrates the quasi–static grasp model proposed in [5] to underactuated hands that includes joints with passive elastic elements, Fig. 1 shows a scheme of a planar underactuated robotic hand, in which the proximal joints are actuated by two tendons, while the distal ones have a passive elastic spring. In [5] the controllability problems that arises when the hand is controlled with a reduced number of actuators was analysed. Hand joints were considered coupled according to a compliant model defined as *soft synergies* [16].

In this paper we introduce the possibility to have some uncontrolled, or passive joints in the hand. The quasi–static model is adopted to evaluate and analyse grasp stiffness. Grasp stiffness was numerically evaluated for some simple planar grippers with different actuation types.

II. Stiffness in Grasping with underactuated hands

The stiffness of a grasp is defined as the linear relationship between a variation of the wrench applied on an object and the resulting motions.

$$\delta w = K \delta u \tag{1}$$

In this expression, $w \in \Re^6$ is the wrench applied to the grasped object, $u \in \Re^6$ is a vector describing object frame configuration $\{B\}$ with respect to an inertial frame $\{N\}$ and the prefix δ indicates that we are considering a *small* variation of grasp configuration with respect to a reference equilibrium condition. The smallness of the variation allows to *linearize* the kinematic and equilibrium relationships that characterize grasp configuration. Grasp stiffness matrix $K \in \Re^{6\times 6}$ describes force/motion relationships [11], [17].

In the following, we will investigate on grasp properties in order to find an expression for grasp stiffness matrix K, and to analyze the parameters that influence it.

Let n_c be the number of contact points. Contacts may occur at any place on the robotic hand. Let $\{C_i^h\}$ be the reference frame on the i-th contact point, connected to the hand, and $\{C_i^o\}$ the reference frame on the i-th contact point, connected to the object. To model grasp forces between the object and the hand, for each contact point i, we introduce the contact force $\lambda_i \in FC_i$, where $FC_i \subset \Re^{l_i}$, and l_i value depends on the type of contact [18], [19]. For instance, $l_i = 1$ if the friction between contact surfaces is negligible, and then the contact force has only one component normal to the object surface in the contact point, while $l_i = 3$ if we want to model the friction between finger and object contact surfaces.

Introducing the above definition into the static equilibrium equation of the grasped object, we obtain

$$w + G\lambda = 0 \tag{2}$$

where $\lambda = [\lambda_1^{\mathrm{T}}, \cdots, \lambda_{n_c}^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{n_l}$ is a vector containing all the contact forces, whose dimension is $n_l = \sum_{i=1}^{n_c} l_i$, and $G \in \Re^{6\times n_l}$ is the *Grasp matrix*.

Let $\hat{c}_i^o \in \Re^6$ denote the vector describing the position and orientation of $\{C_i^o\}$ relative to $\{B\}$, let $c_i^o \in l_i$ include the $\hat{c}_i^o$ components constrained by the contact model, and let $c^o \in \Re^{n_l}$ collect the constrained components for all the contact points. It is easy to verify that the transpose of the Grasp matrix allows to express the contact frame variations on the object as a function of object configuration variation [19]

$$\delta c^o = G^{\mathrm{T}} \delta u \tag{3}$$

A. Modelling the underactuated hand

Let $q =\in \Re^{n_q}$ be a vector that describes hand configuration. Typically the elements of q vector represent hand joint displacement, a rotation in case of revolute joint, a translation in case of prismatic joint.

To take into account hand link and joint mechanical stiffness, the virtual joint method [12], VJM, is adopted. This approach allows to extend the results obtained assuming a rigid model for the links and adding localized virtual springs in the joints, which takes into account link deformation.

With this approach, the actual value of joint variables q is generally different from the corresponding reference values $q_r \in \Re^{n_q}$, let $\tau \in \Re^{n_q}$ represent joint loads (forces in prismatic joints and torques in revolute joints).

The components of contact point displacements on the hand, constrained by the contact model, and expressed with respect to $\{C_i^o\}$ reference frames, can be evaluated as a function of hand joint variation, i.e.

$$\delta c^h = J \delta q \tag{4}$$

in which $J \in \Re^{n_l \times n_q}$ represents the *hand Jacobian matrix*. More details on the evaluation of hand Jacobian matrix can be found in [19]. It is worth to note that the hand Jacobian matrix depends on both the hand joint configuration q and on object displacement vector u, i.e. $J = J(q, u)$.

In a static equilibrium condition, the contact forces that the hand exchanges with the object is balanced by the joint torques τ

$$\tau - J^{\mathrm{T}}\lambda = 0 \tag{5}$$

In this paper we propose a quasi static model that can be applied both to underactuated hands [3] and to fixed motion coupled hands [20].

Firstly we suppose that the hand joint reference configuration q_r can be defined using a number of inputs whose dimension is lower than the number of hand joints, in other terms the relative motions of hand joints are somehow constrained. So, we can define a vector $z \in \Re^{n_z}$ of *Lagrangian variables*, whose dimension is equal to the number of hand DoFs [4], [8]. In the literature, approaches where the actual joint variables is a linear combination or a function of such Lagrangian variables, [21], [22], have been presented. In this paper the joint displacement aggregation corresponds to a reduced dimension representation of hand movements according to a *compliant* model of joint torques, as introduced in [16].

Secondly, we suppose that the Lagrangian coordinates can be divided as follows:

$$z = [z_a^{\mathrm{T}} \;\; z_p^{\mathrm{T}}]^{\mathrm{T}}$$

with $n_z = n_{za} + n_{zp}$: $z_a \in \Re^{n_{za}}$ represents the *active* or *controlled* input variables, corresponding, for instance, to the actuators, while the remaining variables $z_p \in \Re^{n_{zp}}$ represent the uncontrolled or *passive* variables.

In other terms, the number of DoAs (Degrees of Actuation), n_{za} can be lower than the number of DoFs, n_z, this corresponds to the more general definition of underactuated mechanism [4],[8].

The kinematic analysis of the mechanism allows to express the *reference values* of the joint variables q_r, i.e. the values of the joint variables that would be obtained if the hand structure was perfectly stiff or if there are not external loads, as a function of the Lagrangian coordinates z

$$q_r = f_s(z) \tag{6}$$

where $f_z : \Re^{n_z} \to \Re^{n_q}$ represents the generally non–linear kinematic relationship.

This relationship can be differentiated in order to express the joint displacement variation as a function of the Lagrangian coordinate variation:

$$\delta q_r = S \delta z \tag{7}$$

where $S = \frac{\partial f_z}{\partial z}|_0 \in \Re^{n_q \times n_z}$. In synergy actuated hands, as those presented in [5], matrix S corresponds to the *synergy matrix*, in [20] it is referred to as eigengrasp matrix. It is worth to note that, with respect to those works, in this case matrix S includes also the terms that depends on passive Lagrangian variables.

Often the kinematic analysis of hand mechanism leads to an implicit formulation [23]. In this case a set of algebraic equations is defined, that constraint the elements of the configuration vector q_r

$$\psi_c(q) = 0 \tag{8}$$

where $\psi_c \in \Re^{n_q - n_z}$ represents the constraint algebraic expression. Since the mechanism has n_z DoFs, we can choose a vector of variables $n_z \in \Re^{n_z}$ that is able to fully describe its configuration, n_z algebraic equations are necessary to define the Lagrangian variables

$$z = f_l(q) \tag{9}$$

Equations (8) and (9) can be summarized as

$$\psi(q, z) = 0 \tag{10}$$

where $\psi \in \Re^{n_q}$ [24]. By differentiating eq. (10) we obtain

$$[\Psi_q]\dot{q} + [\Psi_z]\dot{z} = 0 \tag{11}$$

where $[\Psi_q] = \frac{\partial \psi}{\partial q} \in \Re^{n_q \times n_q}$ and $[\Psi_z] = \frac{\partial \psi}{\partial z} \in \Re^{n_q \times n_z}$. If the constraints are independent, matrix $[\Psi_q]$ is not singular and we can express the configuration parameter time derivative as a function of Lagrangian variable time derivatives as

$$\dot{q} = -[\Psi_q]^{-1}[\Psi_z]\dot{z} \tag{12}$$

So, matrix S can be defined, with this type of representation of hand kinematics, as

$$S = -[\Psi_q]^{-1}[\Psi_z]$$

It is furthermore easy to verify [24] the following orthogonality relationship

$$[\Psi_q]S = 0 \tag{13}$$

Matrix S elements depend on hand configuration, and thus on z vector element values, i.e. $S = S(z)$.

The forces and torques applied to the hand by the contacts with the object can be reduced to the Lagrangian variables as

$$\sigma = S^{\mathrm{T}}\tau \tag{14}$$

where $\sigma \in \Re^{n_z}$ represents the forces applied by the contact to the hand expressed in the Lagrangian variable space. In an equilibrium configuration, these forces will be balanced by the actuators actions and by the forces that arise when passive elements like springs are deformed from their reference configuration.

Recalling the partition of Lagrange variables in active and passive variables, z_a and z_b respectively, the generalized forces σ can be either partitioned as

$$\sigma_a = S_a^{\mathrm{T}}\tau \qquad \sigma_p = S_p^{\mathrm{T}}\tau$$

That highlights the contribution to hand equilibrium of the forces generated by the actuators σ_a and of the passive elements σ_p.

B. Compliance

The contact force distribution problem for general grasps with many contacts and few controlled inputs is usually an under determined problem of statics, and to solve the indeterminacy an ideal contact model is not adequate [25]. The problem can be solved introducing a set of *springs* interposed between the fingers and the object at the contact points: a contact force variation $\delta\lambda$ from an initial equilibrium configuration can be expressed as

$$\delta\lambda = K_s(\delta c^h - \delta c^o) = K_s(J\delta q - G^{\mathrm{T}}\delta u) \tag{15}$$

where $K_s \in \Re^{n_l \times n_l}$ is the contact compliance matrix symmetric and positive definite.

As outlined in [11], often the structural stiffness of the links and the controllable servo compliance of the joints have the same order of magnitude of the contact stiffness and thus should be considered [26].

When the hand structure is not perfectly stiff, the actual configuration vector q could be different from the reference one q_r, and their difference depends on the applied load τ and on the hand structure compliance, that is described, in the configuration space, by the compliance matrix C_q. In terms of variation with respect to a reference initial configuration the following relationship holds

$$\delta q_r - \delta q = C_q \delta\tau \tag{16}$$

often the inverse of compliance matrix $K_q = C_q^{-1} \in \Re^{n_q \times n_q}$ is defined to describe hand stiffness. It is clear that, for a perfectly rigid hand structure, $C_q = 0$ and K_q is not defined.

Also for the Lagrangian variables, we assume a compliant model. When the idea of hand synergies was initially applied to robotic hands, a rigid coupling between configuration variables and synergies was considered [20], that can be substantially represented as fixed motion coupled hands. This type of modeling approach presented some problems in the contact managing during grasp analysis and then in further works on this issue a *softly underactuated model* [16] was assumed: the synergies reference values z_{ra} are directly controlled, while their actual values depends on the system stiffness and on the applied load. In such synergy actuated hands the presence of passive joints in the hand structure was not explicitly taken into account, i.e. they assumed that $n_{za} = n_z$ and thus $n_{zp} = 0$.

In this paper we extend the proposed quasi static model in order to include also passive joints. So we consider both active and passive Lagrangian variables.

For the active Lagrange variables, let us assume that the actuators are closed loop controlled in position, so that the input variable that the user control is the reference value of the actuator position, z_{ar}. The generalized actuation forces are then proportional to the difference between the reference and actual actuator position

$$\delta\sigma_a = K_{za}(\delta z_{ra} - \delta z_a) \tag{17}$$

where $K_{za} \in \Re^{n_{za} \times n_{za}}$ is a symmetric and positive definite matrix that defines the actuator stiffness. Matrix K_{za} includes both the compliance of the actuation transmission elements (e.g. tendon elasticity), and the gain of actuator control systems.

Also for the passive variables we assume a compliance model, then the Lagrangian forces corresponding to the passive joints are

$$\delta\sigma_p = -K_{zp}\delta z_p \tag{18}$$

where $K_{zp} \in \Re^{n_{zp} \times n_{zp}}$ is a symmetric and positive definite matrix. Note that, while z_{ra} is actively controlled, z_{pr} is constant, and then $\delta z_{pr} = 0$.

Combining eq. (17) and (18) we obtain

$$\delta\sigma = K_z(\delta z_r - \delta z) \tag{19}$$

where $K_z = \mathrm{Blockdiag}([K_{za}, K_{zp}])$, $\delta z_r^{\mathrm{T}} = [\delta z_{ar}^{\mathrm{T}} \;\; 0^{\mathrm{T}}]^{\mathrm{T}}$, $z^{\mathrm{T}} = [z_a^{\mathrm{T}} \;\; z_p^{\mathrm{T}}]^{\mathrm{T}}$, $\sigma^{\mathrm{T}} = [\sigma_a^{\mathrm{T}} \;\; \sigma_p^{\mathrm{T}}]^{\mathrm{T}}$.

III. Linear analysis of grasp problem with underactuated hands

Starting from the reference configuration, indicated with index $_0$ let's consider a small variation. The hand grasping is here considered as a MIMO system in which the inputs are the active Lagrangian variable reference values z_{ar} (controllable input) and the uncontrollable object wrench w, representing in general the interaction between the object and the environment.

In this work we do not consider inertia effects and assume quasi–static conditions. This assumption is feasible if the applied variation is sufficiently slow and if the system is compliant [3]. Starting from an equilibrium condition, if the system is asymptotically stable, after the superimposition of an input variation, it will tend to a new equilibrium configuration.

If the new equilibrium configuration is sufficiently *near* to the reference one, we can assume that the system can be locally linearised. From the linearization of the object equilibrium equation (2), we obtain:

$$\delta w - G\delta\lambda = 0 \tag{20}$$

It is worth to note that, if the contact frame displacements are expressed with respect to the object reference system, and rolling in contact points is not considered, G matrix is constant [17].

Let us then consider the hand equilibrium equation (5), the joint torque variation $\delta\tau$ can be expressed as

$$\delta\tau = J^{\mathrm{T}}\delta\lambda + K_{\tau,q}\delta q + K_{\tau,u}\delta u \tag{21}$$

where $K_{\tau,q} = \frac{\partial J\lambda_0}{\partial q}$ and $K_{\tau,u} = \frac{\partial J\lambda_0}{\partial u}$ take into account the variation of hand Jacobian matrix with respect to q and u variations, respectively. If the contact frame displacements are expressed with respect to the object reference frame, the hand Jacobian depends both on hand and on object configuration. $K_{\tau,q} \in \Re^{n_q \times n_q}$ and $K_{\tau,u} \in \Re^{n_q \times n_d}$ elements dimensionally a stiffness and the matrices are usually referred to as *geometric stiffness matrix* [12]. Furthermore, it can be verified that $K_{\tau,q}$ matrix is symmetric [13].

Finally, concerning the relationship between the generalized Lagrangian forces and joint torques, we can express the variation $\delta\sigma$ as follows:

$$\delta\sigma = S^{\mathrm{T}}\delta\tau + K_\sigma \delta z \tag{22}$$

where $K_\sigma = \frac{\partial S\tau_0}{\partial z} \in Re^{n_z \times n_z}$ takes into account the variation of S matrix with respect to hand configuration, also in this case its elements are dimensionally a stiffness.

The linear relationships defined in eq. (20), (21), (22), (15), (16), (19), (7), can be summarized in the following linear system

$$A \begin{bmatrix} \delta\lambda \\ \delta u \\ \delta\tau \\ \delta q \\ \delta\sigma \\ \delta z \\ \delta q_r \end{bmatrix} = \begin{bmatrix} \delta w \\ 0 \\ 0 \\ 0 \\ 0 \\ \delta z_r \\ 0 \end{bmatrix} \tag{23}$$

with

$$A = \begin{bmatrix} -G & 0 & 0 & 0 & 0 & 0 & 0 \\ J^{\mathrm{T}} & K_{\tau,u} & -I & K_{\tau,q} & 0 & 0 & 0 \\ 0 & 0 & S^{\mathrm{T}} & 0 & -I & K_{\sigma,z} & 0 \\ C_s & G^{\mathrm{T}} & 0 & -J & 0 & 0 & 0 \\ 0 & 0 & -I & -K_q & 0 & 0 & K_q \\ 0 & 0 & 0 & 0 & C_z & I & 0 \\ 0 & 0 & 0 & 0 & 0 & S & -I \end{bmatrix} \tag{24}$$

The system is composed of $n_i = n_d + 3n_q + 2n_z + n_l$ equations, the left hand matrix is square. If it is not singular, it is possible to find the value of the unknowns for a given synergy reference variation δz_r and/or for a uncontrollable variation of the external load δw.

For the stiffness analysis, we need to evaluate the variation of object position δu when an external wrench variation δw is applied to the object, keeping constant the reference actuator value, i.e. $\delta z_r = 0$. Solving eq. (23) it is possible, in particular, to evaluate

$$u = C\delta w \tag{25}$$

and then the grasp stiffness matrix will be given by

$$K = C^{-1} \tag{26}$$

Eq. (23) can be solved analytically, however, the complete solution has a quite complex expression and has not been reported here for the sake of simplicity.

In the following examples we will evaluate grasp stiffness matrix for some simple planar grippers and will analyse its dependency on some grasp parameters.

IV. Numerical examples

A. A four DoFs planar gripper

As an example, let us consider a simple gripper like the one shown in Fig. 2.

The gripper is planar and has two fingers, each finger is composed of two phalanges with the same lengths: the gripper has then 4 DoFs. Let $J_1, \cdots, J_4$ the joint axis traces on the gripper plane, and let $\theta_1, \cdots, \theta_4$ be the joint angles.

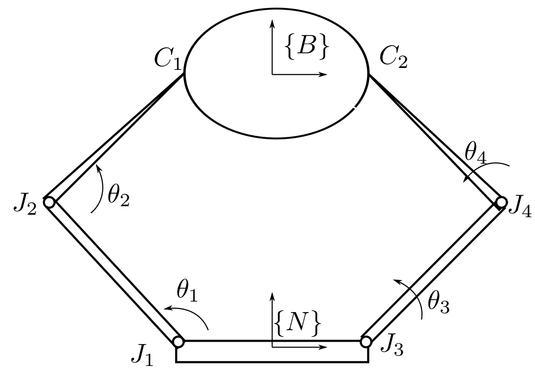

Fig. 2: A four DoFs planar gripper grasping an object, two Hard Finger contact points are present.

The gripper is grasping an object with its fingertips, the contact points are C_1 and C_2, the origin of the local object reference frame is on the mean point of C_1C_2 segment, and the local x axis is parallel to C_1C_2 direction.

The contact model assumed in this test is the hard finger. The object displacement is defined with respect to the base reference system by the vector $u = [u_x \quad u_y \quad \phi]^{\mathrm{T}}$, where ϕ represents the angle between the local and base reference systems abscissae axes.

Indicating with a the length of the finger phalanges, the hand Jacobian matrix is defined as follows

$$J = \begin{bmatrix} J_{1,1} & J_{1,2} & 0 & 0 \\ J_{2,1} & J_{2,2} & 0 & 0 \\ 0 & 0 & J_{3,3} & J_{3,4} \\ 0 & 0 & J_{4,3} & J_{4,4} \end{bmatrix}$$

in which the matrix terms can be expressed as:

$$\begin{aligned} J_{1,1} &= s\phi(ac_{12} + ac_1) - c\phi(as_{12} + as_1) \\ J_{1,2} &= ac_{12}s\phi - as_{12}c\phi \\ J_{2,1} &= s\phi(as_{12} + as_1) + c\phi(ac_{12} + ac_1) \\ J_{2,2} &= as_{12}s\phi + ac_{12}c\phi \\ J_{3,3} &= s\phi(ac_{34} + ac_3) - c\phi(as_{34} + as_3) \\ J_{3,4} &= ac_{34}s\phi - as_{34}c\phi \\ J_{4,3} &= s\phi(as_{34} + as_3) + c\phi(ac_{34} + ac_3) \\ J_{4,4} &= as_{34}s\phi + ac_{34}c\phi \end{aligned}$$

with $s_1 = \sin\theta_1$, $c_1 = \cos\theta_1$, $s_{12} = \sin(\theta_1 + \theta_2)$, and so on. The grasp matrix is given by

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -r & 0 & r \end{bmatrix}$$

where r represents object radius, i.e. the distance between each contact point and the object frame origin.

The geometric terms, that express the variation of J matrix, with respect to q and u, are considered by defining the matrices $K_{J,q}$ and $K_{J,u}$. The matrix that expresses the hand Jacobian derivatives with respect to hand joint variables is given by

$$K_{J,q} = \begin{bmatrix} k_{J,q,1,1} & k_{J,q,1,1} & 0 & 0 \\ k_{J,q,2,1} & k_{J,q,2,2} & 0 & 0 \\ 0 & 0 & k_{J,q,3,3} & k_{J,q,3,4} \\ 0 & 0 & k_{J,q,4,3} & k_{J,q,4,4} \end{bmatrix}$$

in which the matrix terms can be expressed as:

$$
\begin{aligned}
k_{J,q,1,1} &= -\lambda_{01,x}(c\phi(ac_{12}+ac_1)+s\phi(as_{12}+as_1)) \\
&\quad -\lambda_{01,y}(ac_{12}c\phi+as_{12}s\phi) \\
k_{J,q,1,2} &= -\lambda_{01,x}(c\phi ac_{12}+s\phi as_{12}) \\
&\quad -\lambda_{01,y}(ac_{12}c\phi+as_{12}s\phi) \\
k_{J,q,2,1} &= \lambda_{01,x}(s\phi(ac_{12}+ac_1)-c\phi(as_{12}+as_1)) \\
&\quad +\lambda_{01,y}(ac_{12}s\phi-as_{12}s\phi) \\
k_{J,q,2,2} &= \lambda_{01,x}(ac_{12}s\phi-as_{12}c\phi) \\
&\quad +\lambda_{01,y}(ac_{12}s\phi-as_{12}c\phi)
\end{aligned}
$$

and so on, while the matrix that expresses the hand Jacobian derivatives with respect to object displacement is given by

$$
K_{J,u} = \begin{bmatrix} 0 & 0 & k_{J,u,1,3} \\ 0 & 0 & k_{J,u,2,3} \\ 0 & 0 & k_{J,u,3,3} \\ 0 & 0 & k_{J,u,4,3} \end{bmatrix}
$$

in which the matrix terms can be expressed as:

$$
\begin{aligned}
k_{J,u,1,3} &= \lambda_{01,x}(c\phi(ac_{12}+ac_1)+s\phi(as_{12}+as_1)) \\
&\quad +\lambda_{01,y}(ac_{12}c\phi+as_{12}s\phi) \\
k_{J,u,2,3} &= -\lambda_{01,x}(s\phi(ac_{12}+ac_1)-c\phi(as_{12}+as_1)) \\
&\quad -\lambda_{01,y}(ac_{12}s\phi-as_{12}c\phi)
\end{aligned}
$$

($k_{J,u,3,3}$ and $k_{J,u,4,3}$ are similarly defined).

A first numerical analysis was devoted to investigate the effect of geometrical stiffness terms on the total grasp stiffness. We considered a reference configuration in which: $\theta_1 = \frac{3}{4}\pi$ rad, $\theta_2 = -\frac{\pi}{2}$ rad, $\theta_3 = \frac{\pi}{4}$ rad, $\theta_4 = \frac{\pi}{2}$ rad, $w_0 = 0$ N, $\lambda_{01} = [\lambda_0, 0]^{\mathrm{T}}$ N, $\lambda_{02} = [-\lambda_0, 0]^{\mathrm{T}}$ N, where λ_0 value was varied from 0 to 100N, $a = 0.1$ m. Initially the system was considered fully actuated, i.e. $S = I_{4,4}$, where $I_{4,4}$ represents the four dimensional identity matrix. The stiffness matrices were $K_s = k_s I_{4,4}$, $K_q = k_q I_{4,4}$, $K_z = k_z I_{4,4}$ where k_s was expressed in N/m, k_q and k_z in Nm/rad, their numerical values were simultaneously varied from 10 to 1000.

Fig. 3 shows the obtained results in terms of grasp stiffness matrix elements. In these numerical simulations, both the stiffness values and the initial contact force values were varied, in order to investigate the weight of geometrical terms in the overall system stiffness. As it could be expected, for high stiffness values, e.g. $k_s = 1000$ N/m, $k_z = k_q = 1000$ Nm/rad, the effect of contact force variation is negligible: the diagonal terms k_{xx} and k_{yy} are substantially constant, and the extra-diagonal terms k_{xy} and k_{yx} have much lower values. The extra-diagonal terms increase linearly as λ_0 increases, and, for high stiffness values, e.g. $k_s > 100$ N/m, $k_z = k_q > 100$ Nm/rad, their values are substantially independent from system stiffness values. As the stiffness decreases, e.g. $k_s = 10$ N/m, $k_z = k_q = 10$ Nm/rad, the effect of contact force variation becomes significant and either a discontinuity in the overall grasp stiffness terms appears.

B. A four DoFs gripper actuated with two tendons and two passive joints

Let us assume that each finger is driven by a tendon connected to the second joint, where a furthermore a passive

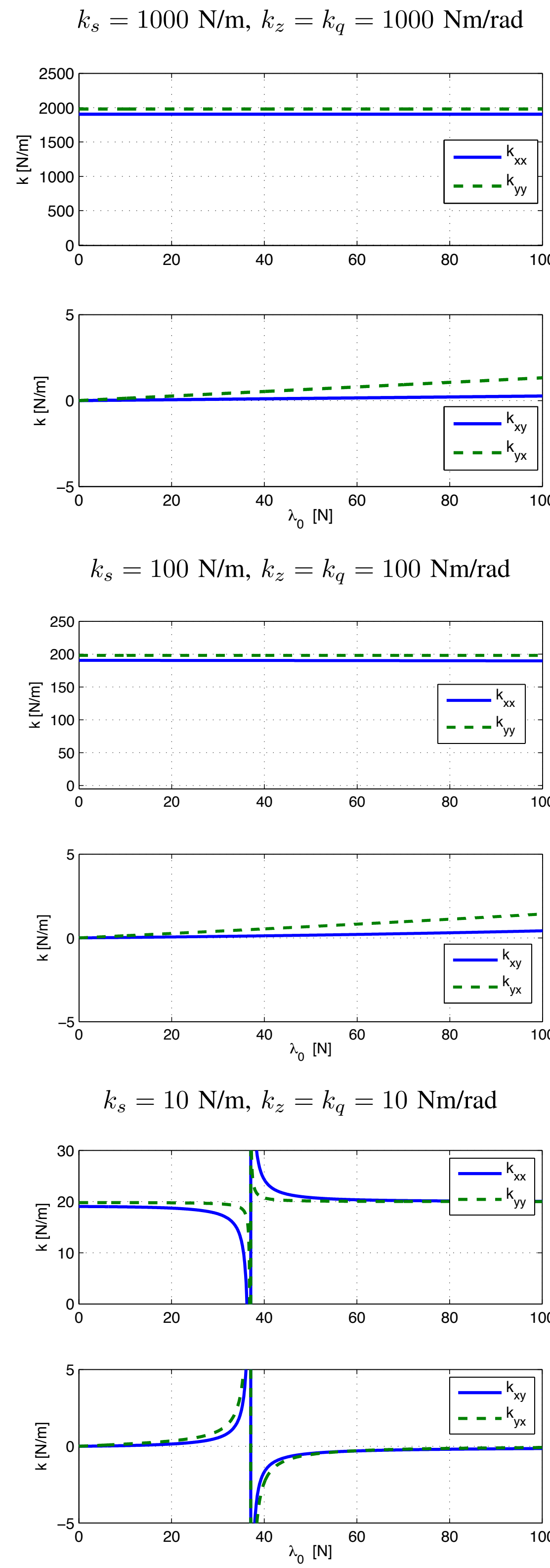

Fig. 3: Linear stiffness terms k_{xx}, k_{yy}, k_{xy}, k_{yx}, for different system compliance values, and different contact force values.

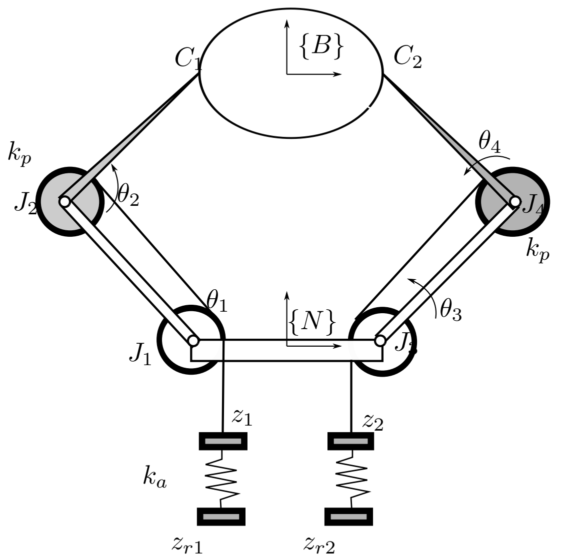

Fig. 4: Example of an underactuated four DoFs gripper, actuated by two elastic tendons, with two passive elastic elements in the joints J_2 and J_4.

elastic element is present, as shown in Fig. 4. This actuation system is a simplified illustrative example inspired by the work presented in [9]. The system has still four degrees of freedom, but only two actuators. We can define the Lagrangian variable vector as $z = [z_a z_b q_2 q_4]^{\mathrm{T}}$, in which z_a and z_b represents tendon displacements and are actively controlled, while q_2 and q_4 are passive. From the kinematic analysis of the mechanism it results that

$$S = \begin{bmatrix} \frac{1}{r_1} & 0 & -\frac{r_1}{r_2} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{r_1} & 0 & -\frac{r_1}{r_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where r_1 and r_2 are the pulley radii on the joints $J_1 - J_3$ and $J_2 - J_4$, respectively. It is worth to note that in this case S is constant and then $K_{S,z} = 0$.

The stiffness matrix K_z is defined in this case as:

$$K_z = \begin{bmatrix} k_a & 0 & 0 & 0 \\ 0 & k_a & 0 & 0 \\ 0 & 0 & k_p & 0 \\ 0 & 0 & 0 & k_p \end{bmatrix}$$

In this case, we analysed the translational part of the grasp stiffness matrix varying some stiffness parameters. In particular, first we varied k_p and then k_a keeping all the other parameters constant. The obtained results are shown in Fig. 5.

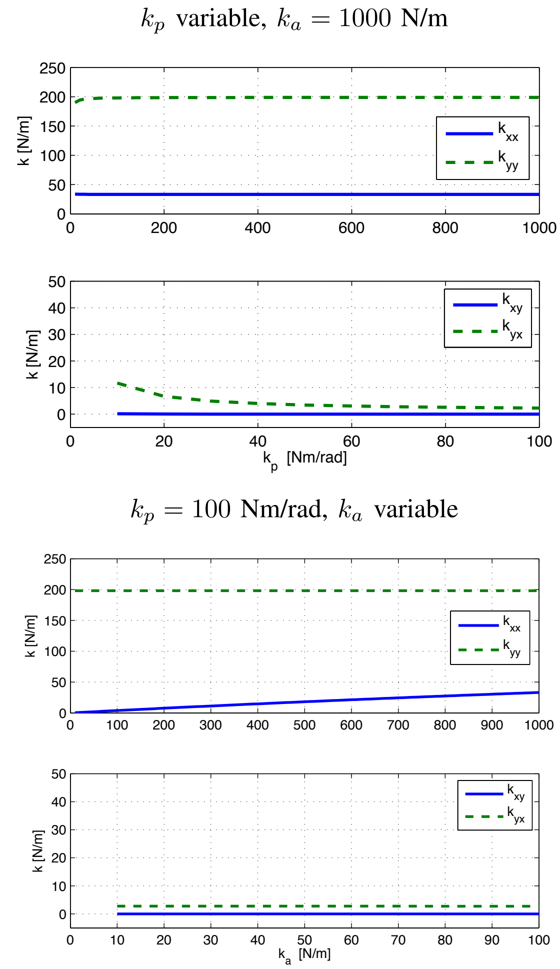

Fig. 5: Linear stiffness terms k_{xx}, k_{yy}, k_{xy}, k_{yx}, for different active and passive stiffness values.

V. Conclusions

The paper presents a quasi-static model of an underactuated compliant robotic hand grasping an object. The proposed model allows to analyse underactuated hands in which some of the DoFs are not actively controlled. The role of compliance is analysed in particular and different compliance sources are included in the model. From the quasi-static problem solution, the grasp stiffness is evaluated, it depends on grasp geometrical parameters, on the system compliance and on the applied force. The grasp stiffness is evaluated for a simple four DoFs gripper. Firstly we evaluated the effect on the grasp stiffness of a variation on the grasp force, that influences the geometrical stiffness terms (depending on hand Jacobian derivatives). For the analysed grasp configuration, we verified that geometrical effects are negligible in a stiff structure, and becomes significant when the hand is very compliant. Then we considered an underactuated solution and analysed grasp stiffness matrix elements for different values of stiffness in the passive joints and in the actuators.

The proposed model allows to find a relationship between grasp stiffness and the main structural and control parameters of the hand, then it could be adopted as a design tool to dimension and optimize such parameters (e.g. passive joint compliance values).

References

[1] A. Bicchi, "Hands for dextrous manipulation and powerful grasping: a difficult road towards simplicity," pp. 2–15, 1996.

[2] J. Butterfass, M. Grebenstein, H. Liu, and G. Hirzinger, "DLR-hand II: next generation of a dextrous robot hand," in *Robotics and Automation, 2001. Proceedings 2001 ICRA. IEEE International Conference on*, vol. 1, 2001, pp. 109–114.

[3] A. Dollar and R. D. Howe, "Joint coupling and actuation design of underactuated hands for unstructured environments," *Mechanics of Structures and Machines*, vol. tocheck, p. tocheck, 2007, check this reference and complete the missing data.

[4] L. Birglen, T. Lalibertè, and C. Gosselin, *Underactuated Robotic Hands*, ser. Springer Tracts in Advanced Robotics, B. Siciliano, O. Khatib, and F. Groen, Eds. Springer, 2008, vol. 40.

[5] D. Prattichizzo, M. Malvezzi, and A. Bicchi, "On motion and force controllability of grasping hands with postural synergies," in *Proceedings of Robotics: Science and Systems*, Zaragoza, Spain, June 2010.

[6] M. Santello, M. Flanders, and J. F. Soechting, "Postural hand synergies for tool use," *The Journal of Neuroscience*, vol. 18, no. 23, pp. 10 105–10 115, December 1998.

[7] L. Birglen and C. M. Gosselin, "Kinetostatic analysis of underactuated fingers," *IEEE Transaction on Robotics and Automation*, vol. 20, no. 2, pp. 211 – 221, 2004.

[8] S. Krut, V. Bégoc, E. Dombre, and F. Pierrot, "Extension of the form closure property to underactuated hands," *IEEE Transactions on Robotics*, vol. 26, pp. 853–866, 2010.

[9] L. U. Odhner and A. Dollar, "Dexterous manipulation with underactuated robotic hands," in *Proc. IEEE Int. Conf. on Robotics and Automation*, Shanghai. China, 2011.

[10] S. Krut, "A force isotropic underactuated finger," in *Proc. IEEE Int. Conf. on Robotics and Automation*, Barcelona, Spain, 2005.

[11] M. Cutkosky and I. Kao, "Computing and controlling the compliance of a robotic hand," *IEEE Transaction on Robotics and Automation*, vol. 5, no. 2, pp. 151–165, 1989.

[12] A. Pashkevich, K. A., and D. Chablat, "Enhanched stiffness modeling of manipulators with passive joints," *Mechanism and Machine Theory*, no. 46, pp. 662–679, 2011.

[13] S. Chen and I. Kao, "Conservative congruence transformation for joint and cartesian stiffness matrices of robotic hands and fingers," *The international Journal of Robotics Research*, vol. 19, no. 9, pp. 835–847, sep 2000.

[14] S. Chen, Y. Li, and I. Kao, "A new theory in stiffness for dextrous manipulation," in *Proc. IEEE Int. Conf. on Robotics and Automation*, Seoul, Korea, 2001.

[15] D. Aukes, B. Heyneman, V. Duchaine, and M. R. Cutkosky, "Varying spring preloads to select grasp strategies in an adaptive hand," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, San Francisco, US, 2011, pp. 1–7.

[16] M. Gabiccini, A. Bicchi, D. Prattichizzo, and M. Malvezzi, "On the role of hand synergies in the optimal choice of grasping forces," *Autonomous Robots*, pp. 1–18.

[17] M. Gabiccini, E. Farnioli, and A. Bicchi, "Grasp and manipulation analysis for synergistic underactuated hands under general loading conditions," in *Proc. IEEE Int. Conf. on Robotics and Automation*, St Paul, Minnesota, USA, 2012.

[18] D. Prattichizzo and J. Trinkle, "Grasping," in *Handbook on Robotics*, B. Siciliano and O. Kathib, Eds. Springer, 2008, pp. 671–700.

[19] R. Murray, Z. Li, and S. Sastry, *A mathematical introduction to Robotic Manipulation*, C. Press, Ed., 1994.

[20] M. T. Ciocarlie and P. K. Allen, "On-line interactive dexterous grasping," in *Haptics: Perception, Devices and Scenarios*, ser. LNCS, M. Ferre, Ed., vol. 5024. Springer, 2008, pp. 104–113.

[21] C. Y. Brown and H. H. Asada, "Inter-Finger Coordination and Postural Synergies in Robot Hands via Mechanical Implementation of Principal Components Analysis," in *2007 IEEE/RSJ International Conference on Intelligent Robots and System*, 2007, pp. 2877–2882.

[22] M. Ciocarlie, C. Goldfeder, and P. Allen, "Dimensionality reduction for hand-independent dexterous robotic grasping," in *Intelligent Robots and Systems, 2007. IROS 2007. IEEE/RSJ International Conference on*, 2007, pp. 3270–3275.

[23] J. Yen, E. J. Haug, and T. O. Tak, "Numerical Methods for Constrained Equations of Motion in Mechanical System Dynamics," *Mechanics of Structures and Machines*, vol. 19, pp. 41–76, 2010.

[24] F. C. Eich-Soellner E., *Numerical methods in multibody dynamics*. B.G. Teubner, 1998.

[25] A. Bicchi, "On the problem of decomposing grasp and manipulation forces in multiple whole-limb manipulation," *Int. Journal of Robotics and Autonomous Systems*, vol. 13, pp. 127–147, 1994.

[26] M. Ciocarlie and P. Allen, "A design and analysis tool for underactuated compliant hands," in *Intelligent Robots and Systems, 2009. IROS 2009. IEEE/RSJ International Conference on*, oct. 2009, pp. 5234 – 5239.