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Internal force control with no object motion in compliant robotic grasps

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Abstract— **The control of internal forces is one of the key issues in grasping. When the robotic hand is compliant, for instance with passive springs at the joints, and the number of controlled variables is low, as shown in recent works for underactuated hands, it is possible that the control of internal forces implies the motion of the manipulated object. This paper deals with this issue and studies the structural conditions for the control of internal forces which do not involve any motion of the grasped object. The analysis is constructive and a controller of internal forces is proposed. Note that guaranteeing zero motion of the object while controlling internal forces is paramount in robotic manipulation when the task requires large accuracy.**

I. INTRODUCTION

This paper deals with robotic hands with compliance. In robotic grasping, compliance can be present at the joint level, at the servo joint control and at the contacts due to soft finger pads or to deformable objects. For a complete characterization refer to [6] where the authors presented a very complete geometrical analysis of compliance in robotic hands. A recent work discussing the relevance of compliance in robotics and its control has been presented in [1] where the authors compared two methods to actively control compliance. Another approach in the literature is to build new robotic underactuated hands with compliant joints able to passively adapt to different grasp configurations which are mainly dominated by the object shape [7] .

Furthermore, it has been shown that in some grasp configurations like whole–arm grasps [2], [10], or in grasps where the hand has a very low number of controlled degrees of freedom (DoF) with respect to the dimension of the contact force vectors, compliance at the contact points allows to solve the problem of force distribution in grasping [12].

In compliant hands statics and kinematics are related by the constitutive equations, that express the contact force as a function of the relative displacements between the contact points on the object and on the hand respectively. This intrinsic relationship between forces and motions implies that the activation, modulation and control of contact forces in general can cause a displacement of the grasped object even the goal is to control only internal, or squeezing forces, as shown in [2].

To better describe the problem of internal force control and object displacements, consider the simple example in Fig. 2. A robotic finger with 3 compliant joints grasps an object through four contact points. Compliance is present only at the joints, i.e. we consider contacts between rigid bodies. The robotic hand is controlled trough the reference joint trajectories q_r.

It can be easily shown that the joint–compliant device in Fig. 2a) is equivalent to the contact–compliant grasp in Fig.. 2b) where the joint variables are q_r and not q and the compliance is present at the contact points only and not at the joints. To get down to the core of the problem, consider the simplest model for the four contacts of the example in Fig. 2: hard finger contact model without friction which transmits only forces along the contact normals. The subspace of internal forces has dimension 2. In particular a base of the internal contact force subspace is given by the contact force vector λ_d and λ_r in Fig. 3: force vector λ_d is obtained by a suitable change of the reference variable $q_{3,r}$ while λ_r is obtained simultaneously changing reference value for both joints J_2 and J_3. From a direct inspection of the grasp forces, it is easy to understand that if do not change the position of the first joint, controlling the internal force along direction λ_r or λ_d involve always a displacement of the center of the object along directions y and x, respectively (Fig. 2). These motions of the object correspond to a change of the energy stored in the joint springs which ultimately give rise to the controlled internal force. However, there is a very important difference between internal force directions λ_r and λ_d. If the control of the first joint J_1 is allowed, from Fig. 2, it is possible to control the motion of the object along the y axis and this motion of the object can be used to compensate the displacement caused by the internal force control. In other terms, the displacement of the center of the object due to the internal force control of λ_r can be compensated by a suitable control of joint reference q_1. On the other hand there is no possibility to compensate displacements of the object consequent to a change of the internal force control along direction λ_d. In other terms the two internal force directions, λ_r and λ_d, are structurally different: both of them move the object changing the energy stored in the spring but only the object displacement generated by one of the two directions can be compensated by a suitable control action of the motion of the object. We will refer to these internal force directions as those corresponding to zero object motions. Roughly speaking those forces are the really internal ones since involve no object motion at all.

Abstracting from the example used to state the problem, in this paper we focus on the joint control of internal forces and object motion. The evaluation of controllable subspaces in terms of internal forces and object displacements is paramount for this work and have been previously inves-

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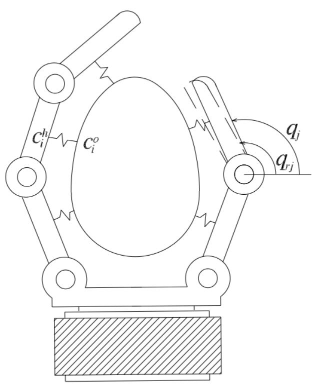


Fig. 1. A generic compliant hand grasping an object.

tigated for whole arms grasps in [13] and for underactuated robotic hands with synergies in [11]. In these papers it has been shown that some coordinated hand/object movements do not involve any change of the elastic energy stored in the grasp compliance, these movements were referred to as rigid body movements. On the other hand, in compliant grasps usually a variation in the contact forces involves a displacement of the object and, since the contact force variation is generated by a deformation of the compliant elements of the grasp (contact springs, joint references, etc.)

In this paper we want to further characterize the set of internal forces in grasping by defining the set of controllable internal forces whose related displacement can be recovered by means of a rigid body motion. The definition of this subset of internal forces is important because they can be controlled and modified in such a way that the object does not move at all. Controlling this type of internal forces prevents undesirable/uncontrolled object movements during grasping operation and then allows to improve the manipulation accuracy and dexterity.

II. Object motion during grasping

A. Static equilibrium equations

In this section we summarizes the equilibrium, congruence and constitutive equations necessary to state the problem. Further details can be found in [12], [14].

Consider a robotic hand grasping an object as those shown in Fig. 1. The force and moment balance for the object can be described by the equation:

$$w = -G\lambda \tag{1}$$

where $w \in \Re^{n_d}$ is the external load wrench applied to the object, whose dimension is $n_d = 6$ for the general three dimensional case and $n_d = 3$ for the simplified planar case; $\lambda \in \Re^{n_l}$ is the contact force vector; $G \in \Re^{n_d \times n_l}$ is the grasp matrix. For a complete definition of the grasp matrix G, the reader is referred to [12]. The solution for the contact forces can be expressed as:

$$\lambda = -G^{\#} w + A\xi \tag{2}$$

where $G^{\#}$ is the pseudoinverse of grasp matrix, $A \in \Re^{n_l \times h}$ is a matrix whose columns form a basis for the nullspace of G ($\ker(G)$) and $\xi \in \Re^{h}$ is a vector that parametrize the homogeneous part of the solution to eq. (1). The generic homogeneous solution $\lambda_o = A\xi$, represents a set of contact forces whose resultant force and moment are zero. The contact forces included in the nullspace of G matrix are referred to as *internal forces* .

Internal forces play a key role in grasp control. In force-closure grasps, a convenient control of internal forces guarantees that the whole vector of contact forces complies with contact friction constraints notwithstanding disturbances acting on the manipulated object.

Not all the internal forces can be arbitrarily controlled by the hand, in order to define the subset of controllable internal forces the hand actuation has to be considered. The relationship between hand joint torques $\tau \in \Re^{n_q}$, where n_q is the number of actuated joints, and contact forces is:

$$\tau = J^{\mathrm{T}} \lambda \tag{3}$$

where $J \in \Re^{n_l \times n_q}$ is the hand jacobian matrix. For further details on its definition and computation, the reader is referred to [12]. We observe that in general the problem is not invertible and thus the contact forces λ cannot be arbitrarily controlled ancting on joint torques τ.

The nullspaces of matrices J and G and their transposes have a relevant influence on the behaviour of the manipulation system. For a complete analysis of these subspaces on the grasp properties refer to [12] and therein references.

If the intersection between the null space of J^{T} and the null space of G is not trivial, the system composed of (1) and (3) results to be statically indeterminate (hyperstatic). In other terms it does not admit a unique solution. To solve the problem of force distribution in this case we need to introduce other equations, usually referred to as constitutive equations, that model the system compliance.

B. Hand joints and contact compliance

Starting from an initial equilibrium condition, the contact force variation is expressed as follows

$$\delta\lambda = K_s(\delta c^h - \delta c^o) \tag{4}$$

where δc^h and δc^o are the displacement of the contact points on the hand and on the object respectively, while $K_s \in \Re^{n_l \times n_l}$ represents the contact stiffness matrix. The contact point displacement on the hand can be related to joint variable variation δq:

$$\delta c^h = J\delta q \tag{5}$$

and contact point displacement on the object can be related to object displacement δu:

$$\delta c^o = G^{\mathrm{T}} \delta u \tag{6}$$

Eq. (7) can be rewritten in terms of *compliance*, taking into account the kinematic relationships (5) and (6) as

$$C_s \delta\lambda = (J\delta q - G^{\mathrm{T}} \delta u) \tag{7}$$

where $C_s = K_s^{-1} \in \Re^{n_l \times n_l}$ represents the contact compliance. Let us then consider the hand joints and assume that they are controlled in position with torque/error static gain $K_q \in \Re^{n_q \times n_q}$ in other terms, the joint torques are proportional to the difference between the reference values of the joint variables, q_r, and their actual values q as $\tau = K_q(q_r - q)$, in terms of variations we can write

$$\delta\tau = K_q(\delta q_r - \delta q) \tag{8}$$

Indicating with $C_q = K_q^{-1} \in \Re^{n_q \times n_q}$ the joint compliance, it can be rewritten as

$$C_q \delta\tau = (\delta q_r - \delta q) \tag{9}$$

Let us then consider in eq. (3) a *small variation* with respect to the reference equilibrium conditions, according to differentiation rules, the following relationship between joint torque and contact force variations can be written

$$\delta\tau = J^{\mathrm{T}}\delta\lambda + (\delta J^{\mathrm{T}})\lambda = J^{\mathrm{T}}\delta\lambda + T\delta q \tag{10}$$

where $T = \left(\frac{\partial J^{\mathrm{T}}}{\partial q}\lambda\right)$ expresses the Jacobian dependency on hand configuration, this term can be neglected if the contact forces λ are *small* in the reference configuration and/or when the hand jacobian does not depend on hand configuration q. In all the other cases it should be considered, and it's effect is all the more relevant as the contact force value is bigger. By substituting eq. (10) in (9), if we assume that $(I + C_q T)$ is invertible, we obtain

$$(I + C_q T)^{-1} C_q J^{\mathrm{T}} \delta\lambda = (I + C_q T)^{-1} \delta q_r - \delta q \tag{11}$$

Let us multiply both the members of (11) by J and sum the result to (7), we obtain

$$\left(C_s + J(I + C_q T)^{-1} C_q J^{\mathrm{T}}\right)\delta\lambda = J(I + C_q T)^{-1}\delta q_r - G^{\mathrm{T}}\delta u$$

that can be rewritten as

$$\delta\lambda = K\left(J_r \delta q_r - G^{\mathrm{T}}\delta u\right) \tag{12}$$

where

$$K = \left(C_s + J_r C_q J^{\mathrm{T}}\right)^{-1} \quad J_r = J(I + C_q T) \tag{13}$$

These expressions for the total hand stiffness matrix are different from those conventionally adopted in grasping theory [6], because they takes into account the jacobian matrix variation.

In [5] the authors showed that taking into account the geometrical effects leads to a transformation between the contact point space and the joint space that preserve the conservative, symmetry and positive definiteness properties of the stiffness matrix.

C. Solution of the quasistatic model

In this section, the subspace of internal forces that can be controlled acting on the joint reference values δq_r is evaluated. According to the procedure described in [2], let us consider an equilibrium starting configuration, in whiche the hand, in the configuration q_0, is grasping an oject on which the external load w_0 is applied, by the contact forces λ_0. The controllable internal force problem has been solved considering, from this initial equilibrium condition, *variation* in the joint variable reference values by δq_r and analysing the new equilibrium condition.

By differentiating eq. (1) and assuming that the external load w_0 is constant, we obtain:

$$0 = -G\delta\lambda + N\delta u \quad N = -\frac{\partial G}{\partial u}\lambda \tag{14}$$

where N takes into account the variation of the grasp matrix elements in the new equilibrium configuration. This term can be neglected only if the grasp matrix is constant and/or the contact forces in the reference configuration are *small*. By substituting expression (12) in eq. (14) we can express the object motion δu as a function of the joint reference variation δq_r as

$$\delta u = \left(GKG^{\mathrm{T}} + N\right)^{-1} GKJ_r \delta q_r = V\delta q_r \tag{15}$$

and the corresponding contact force variation is

$$\delta\lambda_c = \left(I - KG^{\mathrm{T}}\left(GKG^{\mathrm{T}} + N\right)^{-1} G\right) KJ_r \delta q_r = P\delta q_r \tag{16}$$

expressed in terms of the joint activation δq_r.

Let us define $E \in \Re^{n_l \times e}$ as a basis matrix of the column space of matrix P, we can express the generic controllable internal force as

$$\delta\lambda_c = Ey$$

where $y \in \Re^e$ is the generic vector that parametrizes the reachable contact forces.

D. Rigid object motion

Rigid-body kinematics are of particular interest in the control of manipulation systems. They do not involve virtual contact spring deformations, thus they can be regarded as low-energy motions. In fact, rigid-body kinematics represent the easiest way to move the object.

Furthermore, if we impose a rigid body motion to the system, the internal forces do not change. So, when the internal forces are controlled by the joint actuators, we could use the rigid body motion to recover, in some way, the object displacement.

Rigid-body kinematics has been studied in a quasi-static setting in [2], [3] and in terms of unobservable subspaces from contact forces in [9], [10]. In [4] the problem has been analysed also in presence of passive joints.

According to (12), a rigid body motion, i.e. a system displacement that does not involve variation in the contact forces, can be evaluated as a solution of the homogeneous system:

$$J_r \delta q_r - G^{\mathrm{T}}\delta u = 0 \quad \begin{bmatrix} J_r & -G^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \delta q_r \\ \delta u \end{bmatrix} = 0 \tag{17}$$

Rigid kinematics can then be described by a matrix Γ whose columns form a basis for $\ker\begin{bmatrix} J_r & -G^{\mathrm{T}} \end{bmatrix} = \mathrm{im}(\Gamma)$. The generic solution of the system (17) can be expressed as:

$$\begin{bmatrix} \delta q_r \\ \delta u \end{bmatrix} = \Gamma x \tag{18}$$

The matrix Γ can be partitioned as follows:

$$\Gamma = \begin{bmatrix} \Gamma_r & \Gamma_{qc} & 0 \\ 0 & \Gamma_{uc} & \Gamma_i \end{bmatrix}, \tag{19}$$

being Γ_r a basis matrix of the subspace of redundant manipulator motions $\ker(J_r)$, Γ_i a basis matrix of the subspace of indeterminate object motions $\ker(G^T)$, and Γ_{qc} and Γ_{uc} conformal partitions of a complementary basis matrix [1]. It is evident that $J_r\Gamma_{qc} = G^T\Gamma_{uc}$.

The column space of $\Gamma_c = \begin{bmatrix} \Gamma_{qc} \\ \Gamma_{uc} \end{bmatrix}$ consists of coordinated rigid-body motions of the mechanism, for the manipulator (Γ_{qc}) and the object (Γ_{uc}) components. As highlighted before, physically, rigid-body displacements do not involve variation of contact forces.

In [10], it has been shown that rigid-body motions are reachable, i.e. they belong to the space of reachability of the linear system that represents the dynamics of the system, with input the vector of joint generalized forces τ. Notice that the rigid-body subspace is only a subspace of the reproducible motions which also contains motions due to deformations of elastic elements in the model [2].

E. Decoupled force-displacement control

Given a generic object motion that belongs to the rigid body motion subspace, i.e. $\delta u_{rb} = \Gamma_{uc}\beta$, the corresponding set of joint displacements can be evaluated as $\delta q_{rb} = V^{\#}\delta u_{rb} + Q\chi$ where $V^{\#}$ denotes the pseudoinverse of V, Q is a matrix whose columns represents a basis of the nullspace of V, χ is an arbitrary vector whose length is given by the dimension of V nullspace. The corresponding internal force that can be generated is $\delta\lambda_c = PV^{\#}\delta u_{rb} + PQ\chi$.

We can then define the *subspace of internal forces controllable and compensable*: controllable because they can be expressed as $\delta\lambda_c = P\delta q_c$ and thus can be realized acting on the joints, compensable because the corresponding displacement $\delta u_c = V\delta q_c$ belongs to the rigid body motion subspace and thus can be *recovered* with a suitable compensating control action.

For a given internal force variation $\delta\lambda_c$ that belongs to the controllable and compensable subspace, the proposed control for the joint is given by:

$$\delta q_c = \delta q_f + \delta q_m \tag{20}$$

where

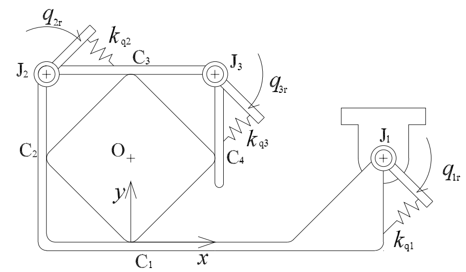
$$\delta q_f = P^{\#}\delta\lambda_c \tag{21}$$

realizes the desired inernal force, while

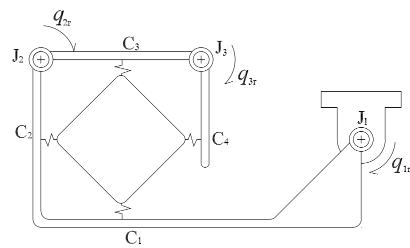
$$\delta q_m = -\Gamma_{qc}\Gamma_{uc}^{\#}V\delta q_f \tag{22}$$

recovers the object motion that would be generated by the control δq_f only. This control strategy allows to change the internal force with zero displacement of the grasped object.

[1] W is called a complementary basis matrix of $\mathscr{V}$ to $\mathscr{X}$ if it is f.c.r. and $\mathrm{im}(W) \oplus \mathscr{V} = \mathscr{X}$.



a)



b)

Fig. 2. Simplified example:the simulated grasp, joints and contact points, the joint compliance has been represented by an *equivalent* contact compliance.

F. Numerical Example

To better explain the main concept of this work, we introduce a simple but paradigmatic example.

Let us consider the planar manipulator in Fig. 2. The hand has three joints, whose axes are J_1, J_2 and J_3, the manipulator is grasping an object, the contact points are C_1, ...,C_4. A reference frame is defined with origin at point C_1 with axes oriented as shown in the figure. The coordinates of the joint axis in the reference configuration, with the previously defined reference system are:

$$J_1 = \begin{bmatrix} 3l \\ l \end{bmatrix} \quad J_2 = \begin{bmatrix} -l \\ 2l \end{bmatrix} \quad J_3 = \begin{bmatrix} l \\ 2l \end{bmatrix}.$$

The contact point coordinates, expressed in the aboved described reference system, are:

$$C_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad C_2 = \begin{bmatrix} -l \\ l \end{bmatrix} \quad C_3 = \begin{bmatrix} 0 \\ 2l \end{bmatrix} \quad C_4 = \begin{bmatrix} l \\ l \end{bmatrix}.$$

The object center is $O = [0, l]^{\mathrm{T}}$.

In this simplified example we consider a hard finger contact model without friction for all contacts. In other terms we suppose that in the contact points no friction is present and the contact force is normal to the contact surface. This simplification is assumed only to simplify and clarify the results of this work to the reader. With this assumption the contact force vector dimension is $\lambda \in \Re^4$ and thus $G \in \Re^{4\times 3}$.

$$G = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We observe that the third row is zero and consequently G^{T} has a non trivial null. Physically this means that the object rotation is not constrained by the contacts, since there is no friction at the contacts. In order to proceed with the algebraic computation, we assume, for the sake of clarity, to disregard any possible rotation of the object. This corresponds to remove the last row of matrix G.

Similarly we can compute the Jacobian matrix $J \in \Re^{4\times 3}$ as

$$J = \begin{bmatrix} -3l & 0 & 0 \\ 0 & 0 & 0 \\ -3l & l & 0 \\ 0 & l & l \end{bmatrix}$$

Compliance matrices C_s and C_q have been chosen so that the total stiffness matrix in (12) results

$$K = kI_{4\times 4}$$

In the following numerical simulations we assumed $k = 1$N/m and $l = 0.1$m. The equivalent manipulator, sketched in Fig. 2 b) where now the joint variables are q_r, the joint are rigid and the equivalent compliance is reported at the contacts.

Controllable internal forces For this simple grasp configuration, we found:

$$E = \begin{bmatrix} -0.05 & 0 \\ -0.05 & -0.05 \\ 0.05 & 0 \\ 0.05 & 0.05 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.0 & 0.05 & 0.05 \\ -0.3 & 0.05 & 0 \end{bmatrix} \quad P = \begin{bmatrix} 0 & -0.05 & 0 \\ 0 & -0.05 & -0.05 \\ 0 & 0.05 & 0 \\ 0 & 0.05 & 0.05 \end{bmatrix}$$

We observe that the activation of joint J_1, δq_{r1} does not imply a contact force variation (since the first column of matrix P is null), while the object displacement, defined by the first column of V matrix, is parallel to y direction. The activation of joint J_2, δq_2 causes a variation in the forces at all the contact points; the object displacement has the same components in the x and y direction. The activation of joint J_3, δq_3 causes a variation of the contact forces at points C_2 and C_4 and an object displacement along the x direction. Fig. 3 shows the internal force variation and the object motion due to activation joints 2 and 3.

We furthermore observe that the change of internal force obtained applying a displacement $\delta q = [0 \quad -\alpha \quad \alpha]^{\mathrm{T}}$ occurs only at contact points C_1 and C_3 and that the corresponding object displacement is in the y direction only as shown in Fig. 4. We furthermore observe that an object displacement in the y direction can be obtained, without producing internal force variation, acting on joint q_1 (see the first column of matrix V).

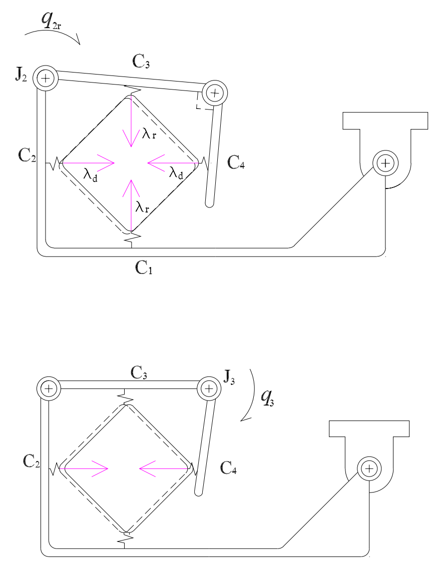


Fig. 3. Force (arrows) and object movement (dashed) variations due to the activation of joint q_2 and q_3 respectively.

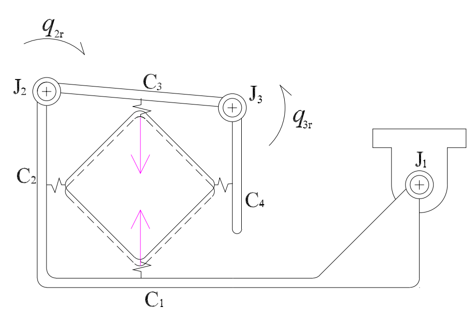


Fig. 4. Force (arrows) and object movement (dashed) variations due to the application of a joint displacement $\delta q = [0 \quad \alpha \quad -\alpha]^{\mathrm{T}}$. The corresponding object displacement is in the y direction and the corresponding internal forces are in the contact points C_1 and C_3.

This result suggests that by properly selecting the joint variables, we can control the *squeezing* action in the y direction *without* moving the grasped object. The same result cannot be obtained for the contact forces on C_2 and C_4.

For this simple grasp configuration, we found that the rigid-body motions are

$$\Gamma_{qc} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \quad \Gamma_{uc} = \begin{bmatrix} 0. \\ 0.3 \end{bmatrix}$$

The rigid body motion is pictorially described in Fig. 5 where it is evident that the rigid body motion is get acting on joint

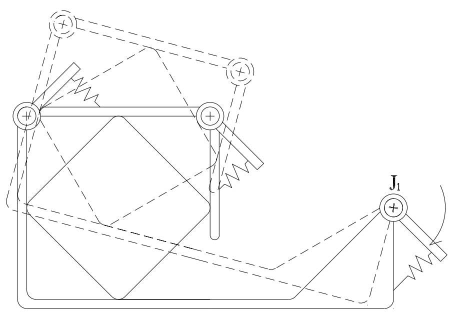


Fig. 5. Rigid body motion for the simulated hand.

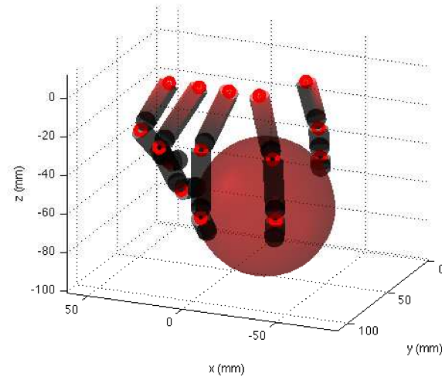


Fig. 6. The human-like hand and the grasped object in the reference configuration.

J_1.

We also observe that the internal force variation whose corresponding object displacement can be compensated is

$$\lambda_c = \begin{bmatrix} -0.039 \\ 0 \\ -0.039 \\ 0 \end{bmatrix} \qquad \delta q_{tot} = \begin{bmatrix} 0.0131 \\ 0.787 \\ -0.787 \end{bmatrix}$$

where δq_{tot} is the total control action to be applied in terms of joint displacements.

III. Application to a human like robotic hand

The analysis of joined control of internal forces and object movements has been applied to a robotic hand with an anthropomorphic kinematic structure, actuated with synergies [11]. The details of the hand kinematic model are described in [8]. The kinematic model of the analysed antropomorphic hand has globally 20 DoFs, four degrees of freedom for each finger. In this paper, a tripod grasp has been discussed, the object (a *sphere*) is grasped with the thumb, index and middle. Each of the three fingers touchs the object only in its tip. A Hard Finger contact model has been considered (single point contact with friction). The layout of the hand and the object is shown in Fig. 6.

The contact force and joint vector dimensions are then $t = 9$ and $n_q = 20$ respectively. Thus for the fully actuated hand $G \in \Re^{6\times 9}$ and $J \in \Re^{9\times 20}$. The dimension of internal force subspace is $h = 3$, the rank of rigid body motion subspace is $\#\Gamma_{uc} = 4$ and the redundancy is $\#\Gamma_r = 13$. The dimension of the controllable internal forces is $\#P = 3$.

In order to reduce the number of controlled joint inputs a synergy based underactuation system has been considered. Problems concerning controllability of internal forces with synergy actuated hands has been presented in [11], where the dimension of controllable internal forces, rigid body motion and redundancy subspaces have been related to the number of activated synergies. In [8], [11] the synergies were furthermore modeled through a compliant structure. In this paper we do not consider such compliance for the sake of simplicity. The proposed results can be easily extended to the more general compliant model.

With this assumption the synergies can be taken into account as follows:

$$\delta q_r = S\delta z \tag{23}$$

where $\delta z \in \Re^{n_z}$ is the variation of the reduced set of inputs and $S \in \Re^{n_q \times n_z}$ is the so-called *synergy matrix*, that maps the input variables z in the joint space q.

The results previously described concerning controllability of internal forces, rigid motions and compensable internal forces can be extended to the synergy actuated hand simply reducing the Jacobian matrix as follows:

$$J_s = JS \tag{24}$$

In the performed simulations, the number of activated synergies has been progressively increased from 1 to 7, the activated synergies have been selected according to their order [8], [11], [15].

Fig. 7 shows the internal force variation $\delta\lambda$ and the corresponding object displacement δu obtained activating one synergy at a time, in this case the first one, i. e. $\delta\lambda = PS\delta z$ where $\delta z = [1, 0, ..., 0, ..., 0]^{\mathrm{T}}$.

In order to verify the results presented in the preceding sections, we analysed, for different numbers of engaged synergies, the dimensions of controllable internal forces $e = \#E$, rigid body motions $\#\Gamma_{uc}$, and controllable and compensable internal forces f.

Increasing the number of engaged synergies from 1 to 3 the number of controllable internal forces increases from 1 to 3, while no rigid motions and consequently no compensable internal forces are possible. If we further increase the number of synergies, the dimension of internal forces increases and consequently we could compensate some components of the object motion. These results are summarized in Table I.

In this example *only the translation motion* of the object have been considered for the compensation of the internal

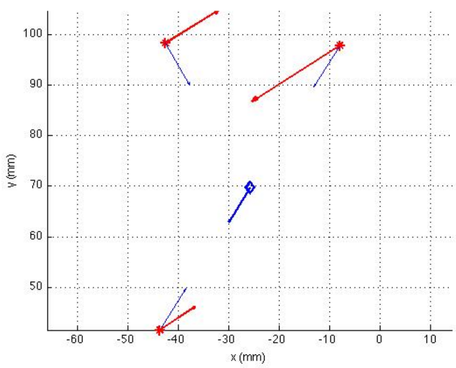


Fig. 7. Internal forces (red arrows) and object displacement (blue arrow) induced by the application of the first synergy, projection on the *xy* plane. The red dots represent the contact points, the blue dot is the object center.

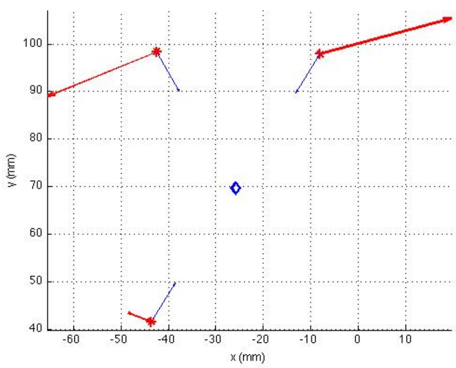


Fig. 8. Example of compensable internal forces, obtained activating 4 synergies; contact point and contact force variations.

forces. In other terms, the rigid body motion has been used to compensate the tranlation motion of the object, while its rotation has not been considered. This example shows that we may choose which motion direction has to be recovered. In other terms, we considered a matrix $\hat{V}$ given by the first three rows of V.

Fig. 8 shows the results obtained using 4 synergies, according to the results shown in Table I, in this case the subspace of controllable internal forces has dimension 3, the rigid body motion subspace has dimension 1, and the subspace of controllable and compensable internal forces has dimension 1.

nz	$\#E$	$\#\Gamma_{uc}$	f
1	1	0	0
2	2	0	0
3	3	0	0
4	3	1	1
5	3	2	2
6	3	3	3
7	3	4	3

TABLE I

HUMAN-LIKE HAND: DIMENSION OF CONTROLLABLE INTERNAL FORCES E_s, RIGID BODY MOTIONS Γ_{uc}, CONTROLLABLE AND COMPLENSABLE INTERNAL FORCESF SUBSPACES AS A FUNCTION OF THE NUMBER OF ACTIVATED SYNERGIES.

IV. CONCLUSIONS

In this paper the control of internal force and object movements in grasping is analysed. Guaranteeing zero motion of the object while controlling internal forces is paramount in robotic manipulation when the task requires large accuracy. On the other hand, when the robotic structure is compliant and/or the number of controlled variables is low, as for instance in underactuated hands, it is possible that the control of internal forces implies the motion of the manipulated object. In this paper, the structural conditions for the control of internal forces which do not involve any motion of the grasped object were analysed. In the paper these forces were defined as *controllable and complensable*, they were identified as a subspace of the internal traditionally defined in grasp theory and a control strategy that allows to tune the contact forces without moving the object was proposed.

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